EEEM034 Exercises — Week 6 solutions

1.

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{c}\quad\quad\quad\pi\end{array} & \\ \left[\begin{array}{c|cc|c} 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 2/2 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] & \eta \end{array}$$

$$B = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} G & B & R \end{array} \\ \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 3/3 & 0 \end{bmatrix} \end{array}$$

2(a)

$\alpha_1(1) = 1 \times \frac{1}{3} = \frac{1}{3}$

$\alpha_1(2) = 0 \times \frac{1}{3} = 0$

$\alpha_2(1) = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{18}$

$\alpha_2(2) = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{18}$

$\alpha_3(1) = \frac{1}{18} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{108}$

$\alpha_3(2) = \left(\frac{1}{18} \times \frac{1}{2} + \frac{1}{18} \times \frac{1}{2}\right) \times \frac{1}{3} = \frac{1}{54}$

$P(O|\lambda) = \frac{1}{54} \times \frac{1}{2} = \frac{1}{108}$

$\beta_3(1) = 0$

$\beta_3(2) = \frac{1}{2}$

$\beta_2(1) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$

$\beta_2(2) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{12}$

$\beta_1(1) = \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{12} + \frac{1}{2} \times \frac{1}{3} \times \frac{1}{12}\right) = \frac{1}{36}$

$\beta_1(2) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{12} = \frac{1}{72}$

$P(O|\lambda) = 1 \times \frac{1}{3} \times \frac{1}{36} = \frac{1}{108}$

(b)

$\gamma_1(1) = 1$

$\gamma_1(2) = 0$

$\gamma_2(1) = \frac{1}{2}$

$\gamma_2(2) = \frac{1}{2}$

$\gamma_3(1) = 0$

$\gamma_3(2) = 1$

$\xi_1(i,j) = 0 \quad \forall i,j$

$\xi_2(1,1) = \xi_2(1,2) = \frac{1}{2}$

$\xi_2(2,1) = \xi_2(2,2) = 0$

$\xi_3(1,1) = 0 \qquad \xi_3(1,2) = \frac{1}{2}$

$\xi_3(2,1) = 0 \qquad \xi_3(2,2) = \frac{1}{2}$

(c)

$$A = \left[\begin{array}{c|cc|c} 0 & 1 & 0 & 0 \\ \hline 0 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1/3 & 2/3 \\ \hline 0 & 0 & 0 & 0 \end{array}\right] \qquad B = \begin{array}{c} \begin{array}{ccc} G & B & R \end{array} \\ \begin{bmatrix} 2/3 & 1/3 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

3. See slide P.11 in HMM Part 5.

4. Univariate:

$$-\ln(b_i(o_t)) = -\ln\left[\left(\frac{1}{(2\pi\Sigma_i)^{1/2}}\right)\exp - \frac{(o_t-\mu_i)^2}{2\Sigma_i}\right]$$

$$= \frac{1}{2}\ln(2\pi\Sigma_i) + \frac{(o_t-\mu_i)^2}{2\Sigma_i}$$

$$= \frac{1}{2}\left[\ln 2\pi + \ln\Sigma_i + \frac{(o_t-\mu_i)^2}{\Sigma_i}\right]$$

Multivariate:

$$-\ln(b_i(\underline{o}_t)) = \frac{1}{2}\left[K\ln 2\pi + \ln|\Sigma_i| + (\underline{o}_t-\underline{\mu}_i)\Sigma_i^{-1}(\underline{o}_t-\underline{\mu}_i)^T\right]$$

5. $P(O|\lambda) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\Sigma_i}} \exp - \frac{(O_t - \mu_i)^2}{2\Sigma_i}$

$$L = \ln(P(O|\lambda)) = \frac{1}{2}\sum_{t=1}^{T}\left[\ln 2\pi + \ln\Sigma_i + \frac{(O_t-\mu_i)^2}{\Sigma_i}\right]$$

(a) $$\frac{\partial L}{\partial \mu_i} = \frac{1}{2}\sum_{t=1}^{T} -\frac{2\mu_i}{\Sigma_i}(O_t-\mu_i) = 0$$

$$\Rightarrow \quad -\frac{\mu_i}{\Sigma_i}\sum_{t=1}^{T}(O_t-\mu_i) = 0$$

$$\Rightarrow \quad \mu_i = \frac{1}{T}\sum_{t=1}^{T} O_t$$

(b) $$\frac{\partial L}{\partial \Sigma_i} = \frac{1}{2}\sum_{t=1}^{T}\left[\frac{1}{\Sigma_i} - \frac{(O_t-\mu_i)^2}{\Sigma_i^2}\right] = 0$$

$$\Rightarrow \quad \frac{1}{2\Sigma_i}\sum_{t=1}^{T}\left[1 - \frac{(O_t-\mu_i)^2}{\Sigma_i}\right] = 0$$

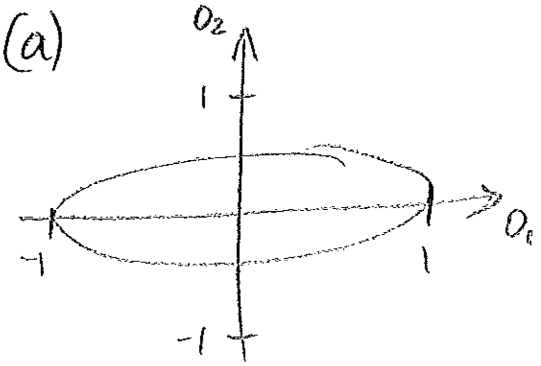
$$\Rightarrow \quad \Sigma_i = \frac{1}{T}\sum_{t=1}^{T}(O_t-\mu_i)^2$$

6 (a) 4 and 9, respectively

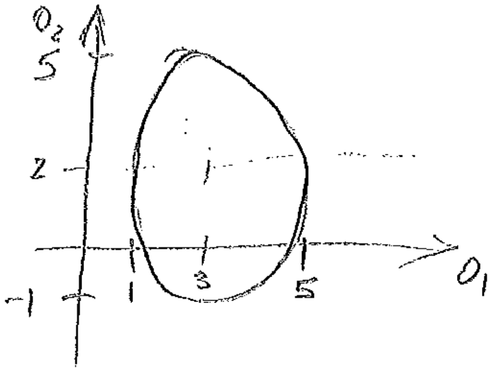
(b) $\Sigma = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$

(c) $|\Sigma| = 4 \times 9 = 36$

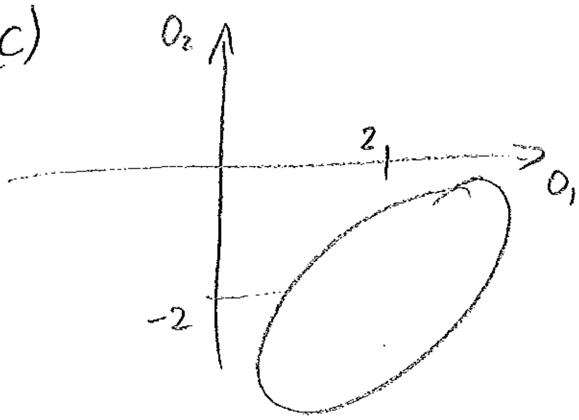
7. (a)



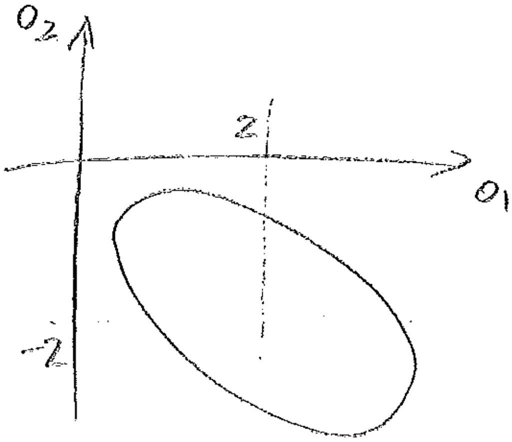
(b)



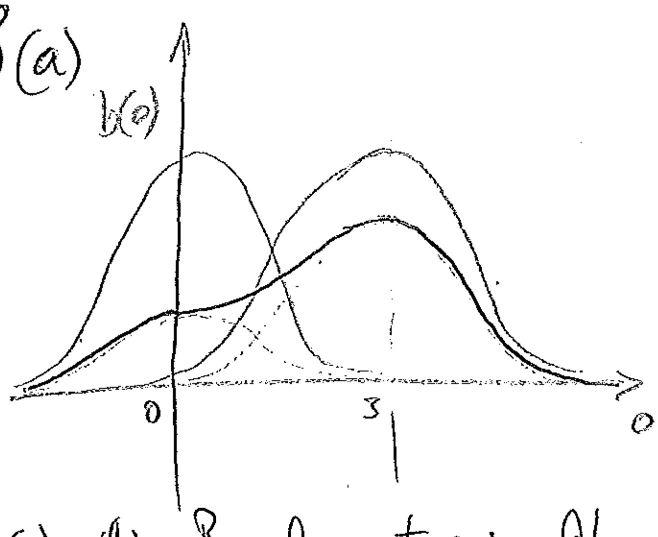
(c)



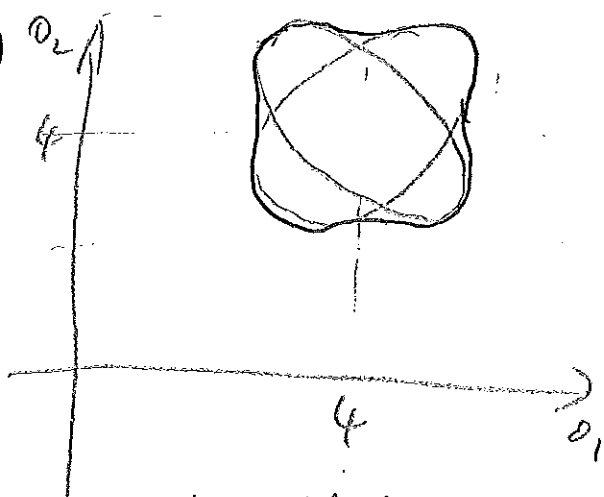
(d)



8 (a)



(b)



9. (a) and (b) Based on training files $r \in \{1..R\}$, extend slides P.8 and P.10.