EE1.el3 (EEE1023): Electronics III

Acoustics lecture 18

Room acoustics

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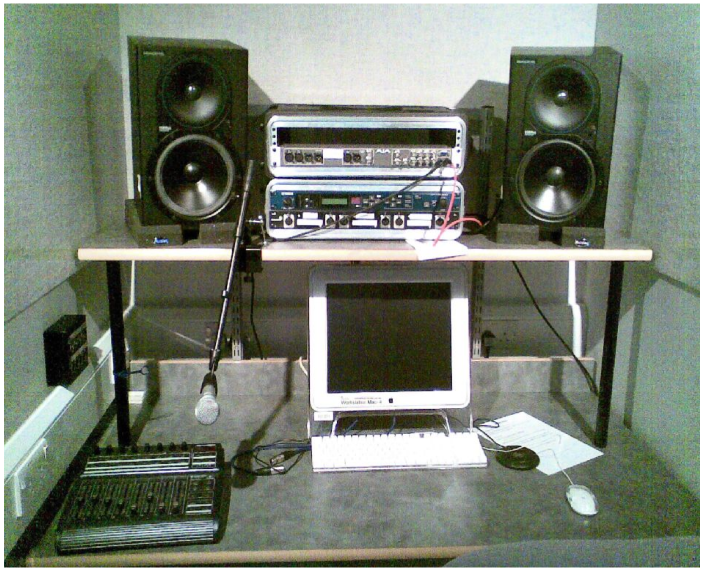

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Room acoustics

Objectives:

- relate sound energy density to rate of absorption
- compare direct and reflected sound levels
- measure and calculate reverberation time
- explain differences between free and diffuse sound fields
- define an expression for a room's critical distance

Topics:

- Direct and reflected sound echoes
- Sound energy stored in a room
- A room's characteristic measurements
- Worked examples

Preparation for room acoustics

- What is the absorption coefficient α?
  - find a definition
  - give two examples of materials with different values of α

- How can we characterise the amount of reflection or absorption in a room?
  - find out what is the meaning of *reverberation time*
  - how can it be measured?

Reflections in a room

Direct sound

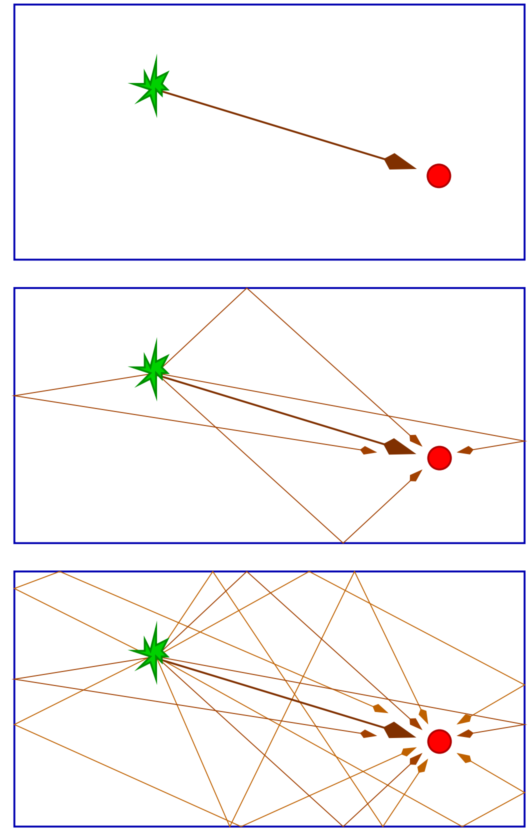
- original waveform from source

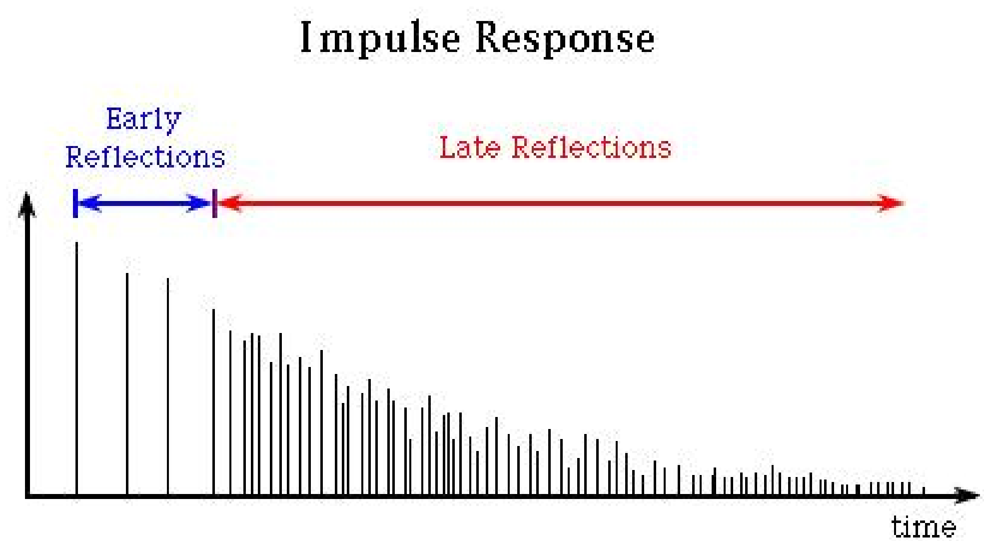
Early reflections

- reinforce direct sound
- inform on room size & shape

Reverberation (late reflections)

- strengthens room impression





Echoes

All surfaces both **ab**sorb and **re**flect sound. The absorption coefficient α gives the ratio of absorbed to **in**cident sound:

$$W_{\text{ab}} = \alpha W_{\text{in}} \qquad\qquad W_{\text{re}} = (1-\alpha) W_{\text{in}}$$

Echoes need delay $>$50 ms to be heard distinctly

Parallel reflective surfaces produce **flutter echoes**:

- large separation $\rightarrow$ fast repetitions
- medium separation $\rightarrow$ buzzing
- small separation $\rightarrow$ ringing

Combination of multiple echoes produces **reverberation**:

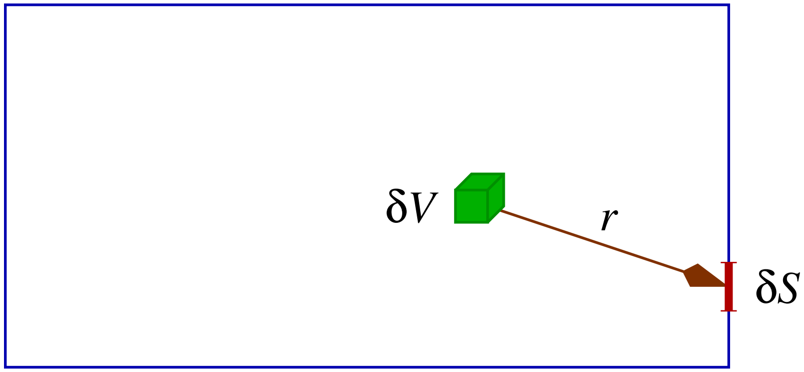
- sound reflected by all surfaces
- from all directions with equal probability
- impulse response exhibits an exponential decay

Diffuse sound stored in a room

Absorption of sound in a room

Rooms provide means of storing and absorbing sound.

By analysing the effect of sound in a small volume, δV, on a small patch of wall, δS, it can be shown that the **intensity** of incident sound depends on the **energy density**, $\mathcal{E}$, in the room:



$$I_{\text{in}} = \frac{\mathcal{E}c}{4} \qquad \text{where} \quad \mathcal{E} = \frac{p_{\text{rms}}^2}{\rho_0 c^2} \tag{1}$$

With surface area S and average absorption α, the rate of energy absorption by the room determines its **absorbed sound power**:

$$W_{\text{ab}} = \frac{S\alpha\,\mathcal{E}c}{4} \tag{2}$$

Calculating sound absorption

Material	Frequency (Hz) 125	250	500	1000	2000	4000
concrete	.01	.01	.02	**.02**	.02	.02
glass	.35	.25	.18	.12	.07	.04
wood	.30	.25	.20	.17	.15	.10
carpet	.02	.06	.14	**.35**	.60	.65
fibreglass	.08	.25	.45	.75	.75	.65
acoustic tile	.40	.50	.60	.75	.70	.60
audience	.40	.55	.80	**.95**	.90	.85

Table 1. Typical sound absorption coefficients α

For a room 4 m (L) × 3 m (W) × 2.5 m (H) with wooden walls, acoustic tiled ceiling and carpet on the floor, we can calculate the total absorption at 1 kHz:

$$A = (2 \times (4+3) \times 2.5 \times .17) + (4 \times 3 \times .75) + (4 \times 3 \times .35)$$

$$= 5.95 + 9.00 + 4.20 = 19.15 \text{ sabin}$$

Growth of sound in a room

From eq. 2, we obtain a differential equation to describe the growth of sound energy within a closed volume V:

$$V\frac{d\mathcal{E}}{dt} = W_{\mathsf{src}} - \frac{S\alpha\,\mathcal{E}c}{4} \tag{3}$$

$$\Rightarrow \qquad \frac{4V}{S\alpha c}\frac{d\mathcal{E}}{dt} + \mathcal{E} = \frac{4W_{\mathsf{src}}}{S\alpha c}$$

If the sound source is started at $t = 0$, solution yields

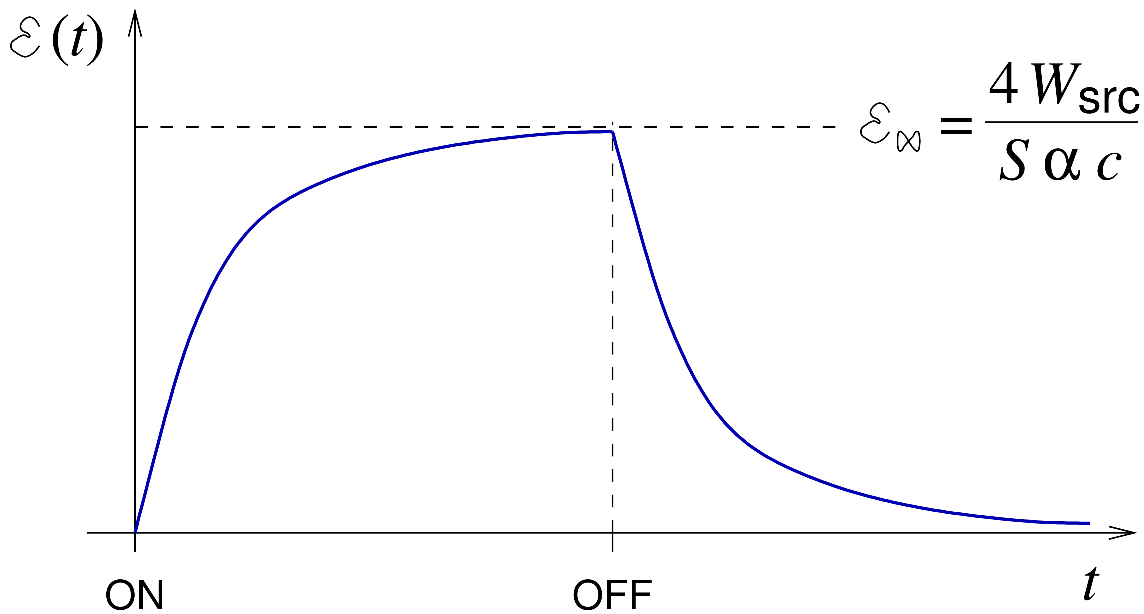
$$\mathcal{E}(t) = \frac{4W_{\mathsf{src}}}{S\alpha c}\left(1 - \mathrm{e}^{-\frac{S\alpha c}{4V}t}\right) \tag{4}$$

As $t \to \infty$, the energy density reaches equilibrium:

$$\boxed{\mathcal{E}^{\infty} = \frac{4W_{\mathsf{src}}}{S\alpha c}} \tag{5}$$

Absorption rate at equilibrium

Energy in a reverberant room acts like a first-order system:



At equilibrium, the rate of energy absorption matches the input power:

$$W_{\text{src}} = W_{\text{ab}}^{\infty} = \frac{S\alpha\,\mathcal{E}^{\infty}\,c}{4} \qquad (6)$$

Rate of reverberation decay

If the source is switched off, eq. 3 gives an expression for the rate of decay of the reverberation:

$$\frac{d\mathcal{E}}{dt} = -\frac{S\alpha\,\mathcal{E}c}{4V} \tag{7}$$

The initial conditions then provide the full equation:

$$\mathcal{E}(t) = \mathcal{E}_0 \exp\left(-\frac{S\alpha c}{4V}t\right) \tag{8}$$

Also, from eq. 1, we relate energy density to sound level:

$$\begin{aligned}
\mathrm{SPL}(t) &= 10\log_{10}\left(\frac{p_{\mathrm{rms}}^2}{p_{\mathrm{ref}}^2}\right) = 10\log_{10}\left(\frac{\mathcal{E}(t)\rho_0 c^2}{p_{\mathrm{ref}}^2}\right) \\
&= 10\left(\log_{10}\mathcal{E}(t) + \log_{10}\rho_0 c^2 - \log_{10} p_{\mathrm{ref}}^2\right) \\
&= 10\log_{10}\mathcal{E}(t) + C \\
&= 10\left(\log_{10}\mathcal{E}_0 - \frac{S\alpha c}{4V}t\log_{10}\mathrm{e}\right) + C \qquad (9)
\end{aligned}$$

Reverberation time

The **reverberation time**, T_{60}, is the time for the SPL in a room to drop by 60 dB, i.e., when $t = T_{60}$ in eq. 9:

$$-10\frac{S\alpha c}{4V}T_{60}\log_{10}\mathrm{e} \;=\; -60$$

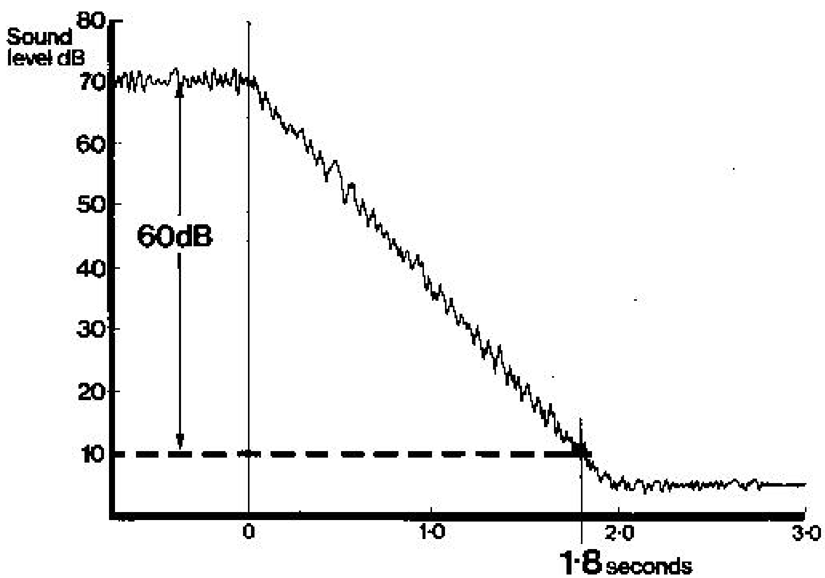
$$\boxed{T_{60} \;=\; \frac{KV}{S\alpha}} \qquad (10)$$

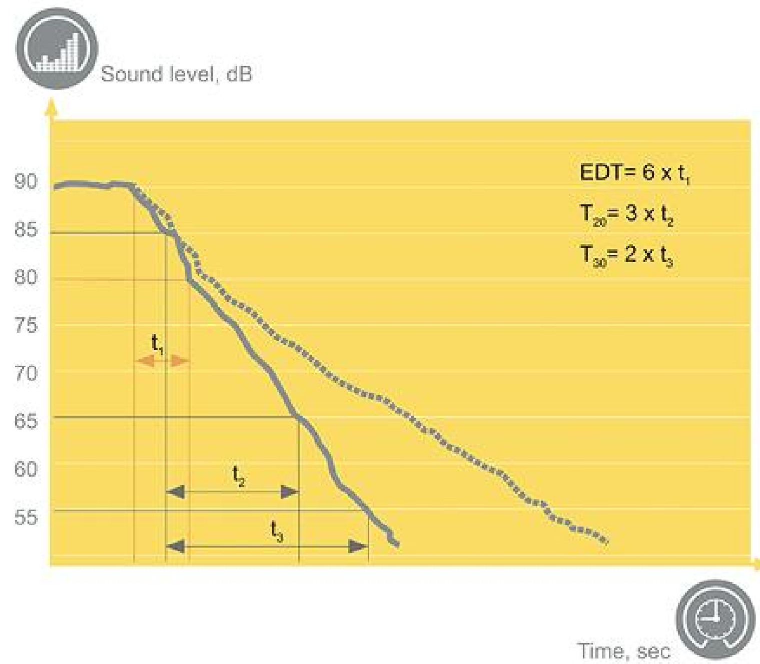
where $K = 24/(c\log_{10}\mathrm{e}) \approx 0.16\,\mathrm{s\,m^{-1}}$ gives Sabine's Eq.

Note: T_{60} can depend on frequency but is otherwise independent of original sound source

Measurement of reverberation time

Reverberation time is commonly measured with pink noise from a loudspeaker:





With limited dynamic range, T_{30} is measured to avoid the noise floor and doubled to give T_{60}. As reverberation time varies with frequency, band-limited noise can be used.

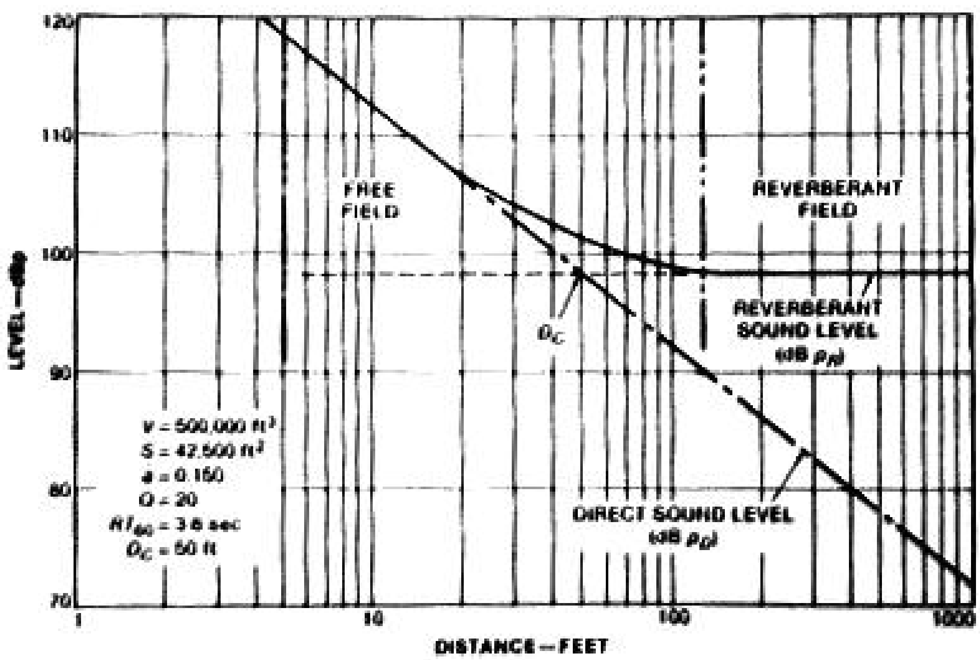
Soundfields in an acoustic environment

Free field, no reflections, sound comes direct from source

Near field (close), more direct sound than reflections

Diffuse field (reverberant field), more reflected sound from all directions than direct sound

Critical distance (a.k.a. room radius), d_c, equal levels of direct and reflected sound



Level of reverberant sound

As in eq. 3, we have the ODE for reverberant energy:

$$
\begin{aligned}
V\frac{d\mathcal{E}_{\text{rev}}}{dt} &= (1-\alpha)\,W_{\text{src}} - \frac{S\alpha\,\mathcal{E}_{\text{rev}}c}{4} \\
\Rightarrow \qquad \frac{4V}{S\alpha c}\frac{d\mathcal{E}_{\text{rev}}}{dt} + \mathcal{E}_{\text{rev}} &= \frac{4\,(1-\alpha)}{S\alpha c}\,W_{\text{src}} \\
\Rightarrow \qquad \mathcal{E}_{\text{rev}}(t) &= \frac{4\,(1-\alpha)\,W_{\text{src}}}{S\alpha c}\left(1-\mathbf{e}^{-\frac{S\alpha c}{4V}t}\right) \\
\Rightarrow \qquad \mathcal{E}_{\text{rev}}^{\infty} &= \frac{4\,(1-\alpha)\,W_{\text{src}}}{S\alpha c}
\end{aligned}
$$

From eq. 1, we can obtain the reverberant SPL:

$$
\begin{aligned}
\text{SPL}_{\text{rev}} &= 10\log_{10}\left(\frac{\mathcal{E}_{\text{rev}}^{\infty}\,\rho_0 c^2}{p_{\text{ref}}^2}\right) \\
&= 10\log_{10}\left(\frac{4\,(1-\alpha)\,W_{\text{src}}\,\rho_0 c}{S\alpha\ p_{\text{ref}}^2}\right) \\
&= 10\log_{10}\left(\frac{4W_{\text{src}}}{R_A I_{\text{ref}}}\right)
\end{aligned}
$$

Critical distance calculation

The critical distance d_c is reached when $\text{SPL}_{\text{rev}} = \text{SPL}_{\text{dir}}$.

For direct sound from a point source in free field, we have

$$\text{SPL}_{\text{dir}} \;=\; \text{SIL}_{\text{dir}} = 10\log_{10}\left(\frac{I_{\text{dir}}}{I_{\text{ref}}}\right) \;=\; 10\log_{10}\left(\frac{W_{\text{src}}}{4\pi d_c^2 I_{\text{ref}}}\right)$$

which matches reverberant sound level at critical distance

$$\text{SPL}_{\text{rev}} \;=\; 10\log_{10}\left(\frac{4W_{\text{src}}}{R_A I_{\text{ref}}}\right)$$

where $R_A = S\alpha/(1-\alpha)$ is the room absorption constant.

Source power W_{src} and reference intensity I_{ref} cancel out, to yield the final result:

$$\frac{4}{R_A} \;=\; \frac{1}{4\pi d_c^2} \qquad \Rightarrow \qquad \boxed{d_c \;=\; \frac{1}{4}\sqrt{\frac{R_A}{\pi}}} \tag{11}$$

Worked example: factory noise

A machine with a sound power of $W_{\text{src}} = 1\,\text{W}$ is situated in a room that has dimensions 7 m (L) × 5 m (W) × 3 m (H).

1. Treating it as a point source, what is the intensity at a distance of 2 m?

2. The machine is left running continuously and reflections contribute to build up a reverberant field. If the absorption coefficient is $\alpha = 0.1$, what will be the asymptotic energy density?

3. What is the corresponding pressure level of the reverberant sound?

4. What changes would occur if the ceiling was treated with a material having $\alpha = 0.6$?

Room acoustics

- Reflection and absorption of sound in a room
- ODE of sound stored in a room
- Measurement of reverberation time
- Calculation of critical distance

Reference

L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders, "Fundamentals of Acoustics", 4th ed., Wiley, 2000. Chapter 12, [shelf 534 KIN]

Preparation for musical acoustics

- For one instrument from each class, identify what creates a sound **source**, and a **resonator** that enhances it?
  - **voice**
  - **woodwind**
  - **brass**
  - **percussion**
  - **string**

Appendix: Sound stored in a room

Absorption of sound in a room

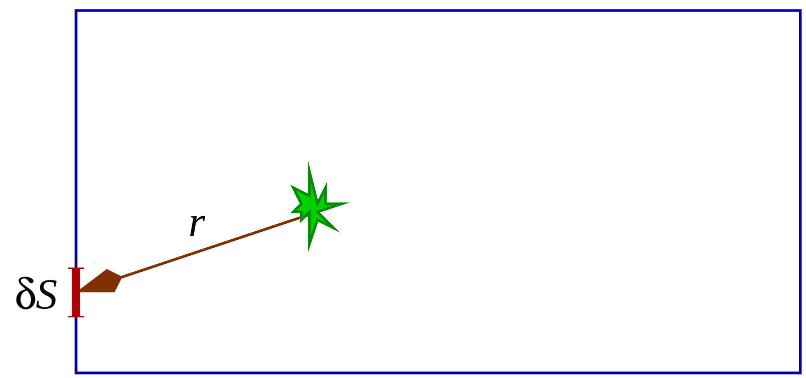
Sound from a compact source impinges on a patch δS and in a reverberant field:

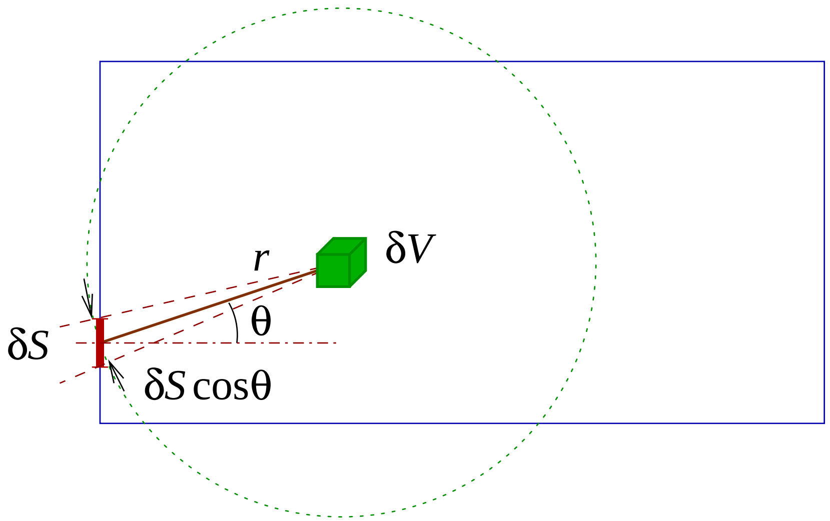
$$\begin{aligned} \delta W &= \delta S \cos\theta \frac{W_{\text{src}}}{4\pi r^2} \\ \delta E &= \delta S \cos\theta \frac{\mathcal{E}\,\delta V}{4\pi r^2} \end{aligned} \tag{12}$$

where **sound energy density** $\mathcal{E}$ is related to the RMS pressure of the diffuse field:

$$\mathcal{E} = \frac{p_{\text{rms}}^2}{\rho_0 c^2}, \tag{13}$$

assuming the constituents act like plane waves (eq. 7a on N.6).



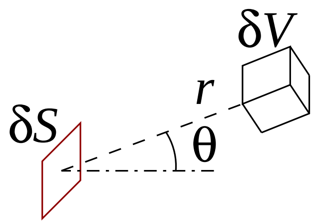


Derivation of the reverberant energy density

Considering a hemispherical shell of thickness δr and radius r centred on δS, we first obtain the energy in a ring:

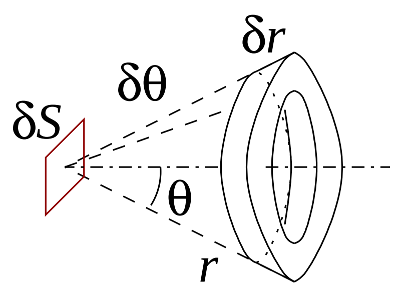
$$\delta V = 2\pi r \sin\theta \, \delta r \, r\delta\theta$$

The **total energy** in time interval $\delta t = \delta r / c$ is:



$$E = \int dE = \int \delta S \cos\theta \, \frac{\mathcal{E}}{4\pi r^2} \, dV$$

$$= \delta S \mathcal{E} \int_0^{\pi/2} \cos\theta \, \frac{2\pi r \sin\theta \, \delta r \, r}{4\pi r^2} \, d\theta$$



$$= \frac{\delta S \, \mathcal{E} \, \delta r}{2} \int_0^{\pi/2} \cos\theta \, \sin\theta \, d\theta$$

$$= \frac{\delta S \, \mathcal{E} \, \delta r}{4} \int_0^{\pi/2} \sin 2\theta \, d\theta$$

$$= \frac{\delta S \, \mathcal{E} \, \delta r}{4} \left[-\frac{1}{2} \cos 2\theta \right]_0^{\pi/2} = \frac{\delta S \, \mathcal{E} \, \delta r}{4} \quad (14)$$

Surface intensity from a diffuse field

From the total sound energy on a small patch of the wall

$$E = \frac{\delta S \, \mathcal{E} \, \delta r}{4}$$

the rate of incident energy per unit area gives the intensity

$$I_{\text{in}} \; = \; \frac{E}{\delta t \, \delta S} \; = \; \frac{\mathcal{E} c}{4} \tag{15}$$

The result is 1/4 of that for a normally-incident plane wave:

$$I_{\perp} = \mathcal{E} c$$

With total sound absorption of surfaces $A = \int \alpha \, \delta S = S\alpha$ with area S and average absorption α, the rate of energy absorption defines the **absorbed sound power**:

$$W_{\text{ab}} = \frac{S\alpha \, \mathcal{E} c}{4} \tag{16}$$