EE1.el3 (EEE1023): Electronics III

Acoustics lecture 16
Standing waves & room modes

Dr Philip Jackson

www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3

UNIVERSITY OF SURREY

Acoustic modes

Objectives:

- derive mathematical expressions for soundfield in space
- identify the modes of standing waves
- relate room dimensions to standing waves

Topics:

- Soundfields in cavities
- Standing waves
- Reverberation

Preparation for Acoustic modes

- What is the difference between 'early' reflections and reverberation?
- What is the 'critical distance' of an acoustical environment?
  - find a definition
  - give an example

- What is a standing wave?
  - find a definition
  - give an example

Acoustics applications

Related applications:

- room, concert hall and studio acoustics
- sound reinforcement, PA and venue design
- noise treatment, suppression of flutter
- noise and vibration from duct resonances
- noise in workplace: factory, restaurant, bar
- design of musical instruments
- loudspeaker and hi-fi design
- mobile audio/multimedia devices

General concepts:

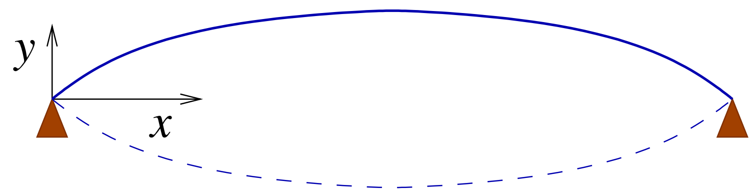
- reflected waves and resonant modes
- energy density within a diffuse field

Resonance

In a bounded space, **resonance** occurs when waves and reflections travelling in opposite directions constructively interfere. The result is a **standing wave** that appears to stand still with a specific **mode shape**:

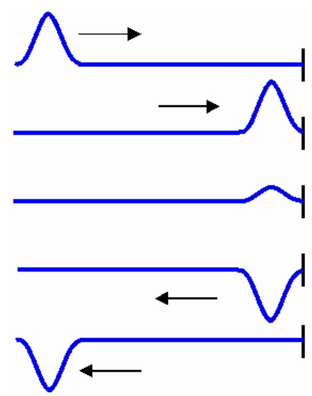
$$y(t,x) = \text{Re}\left\{X(x)\,e^{j\omega t}\right\} = X(x)\,\cos\omega t \tag{1}$$

On a string, the fixed ends reflect the transverse waves in anti-phase.



Thus, the first resonance occurs when the period equals the return path's travel time, $T = 2L/v$:

$$f = \frac{v}{2L}$$



Mode shapes on a string

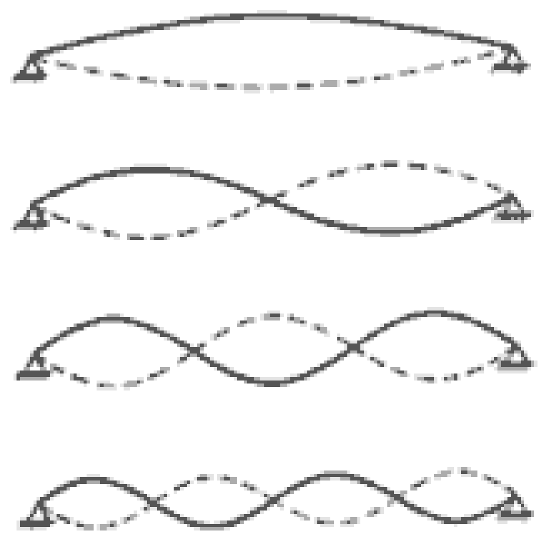
Resonance occurs at any frequency where the positive and negative travelling waves combine to satisfy the boundary conditions, $y(t, 0) = 0$ and $y(t, L) = 0$:

$$f = \frac{v}{2L}$$

$$f = \frac{v}{L}$$

$$f = \frac{3v}{2L}$$

$$f = \frac{4v}{2L}$$



The mode shape for each mode number, $n \in \{1, 2, 3, 4, \ldots\}$, has the form

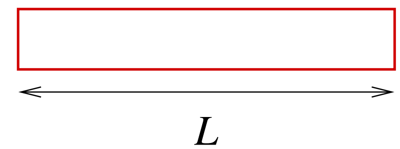
$$X(x) = \sin \frac{\pi n x}{L} \tag{2}$$

Resonance in a cavity

Sound waves produce acoustic resonance in a space with reflecting boundaries:

$$p(t,x) = \text{Re}\left\{X(x)\,\mathbf{e}^{j\omega t}\right\} = X(x)\,\cos\omega t \qquad (3)$$

In a closed pipe, boundary conditions allow sound pressures but prevent any motion at the ends, i.e., zero acoustic velocity, $u(t,0) = 0$ and $u(t,L) = 0$.



By Newton's 2nd law, for there always to be zero velocity at those points, there must also be no pressure gradient:

$$\left(\frac{\partial p}{\partial x}\right)_{x=0} = 0 \qquad \text{and} \qquad \left(\frac{\partial p}{\partial x}\right)_{x=L} = 0$$

The mode shapes are solutions to the 1-D wave equation that satisfy these conditions:

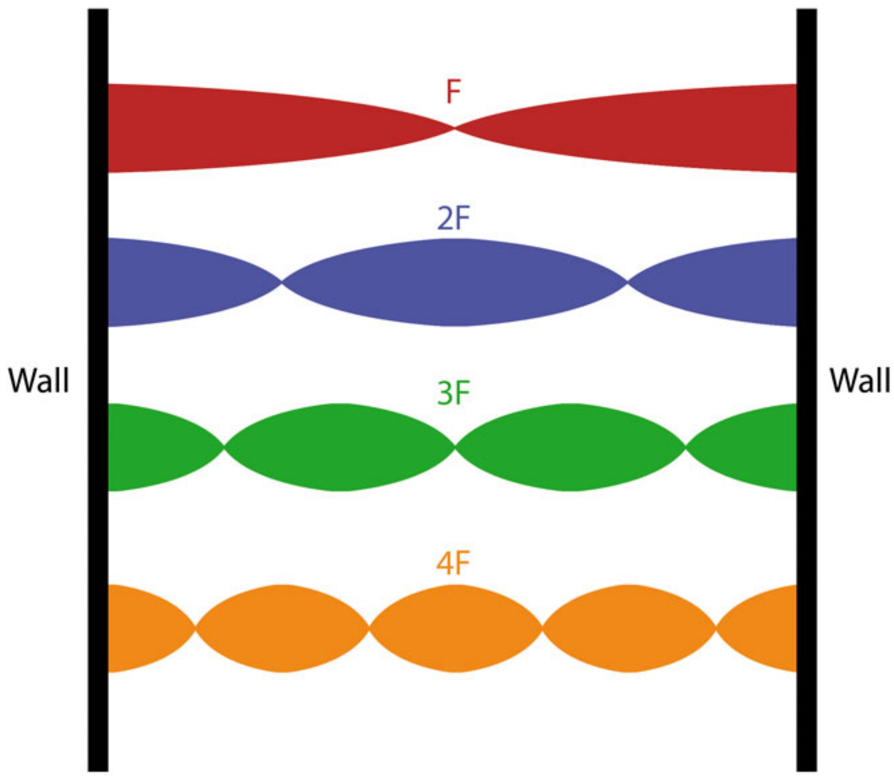
$$X(x) = \cos\frac{\pi n x}{L} \qquad (4)$$

Mode shapes in a cavity

Resonances occur when the return path is a multiple of the wavelength, e.g., length L is whole no. of $\frac{1}{2}$ wavelengths,

$$\lambda = \frac{2L}{n} \qquad \text{or} \qquad f = \frac{nc}{2L}$$

for mode numbers $n \in \{1, 2, 3, 4, \ldots\}$



Amplitude of variation for pressure modes: showing **nodes** and **antinodes** at walls.

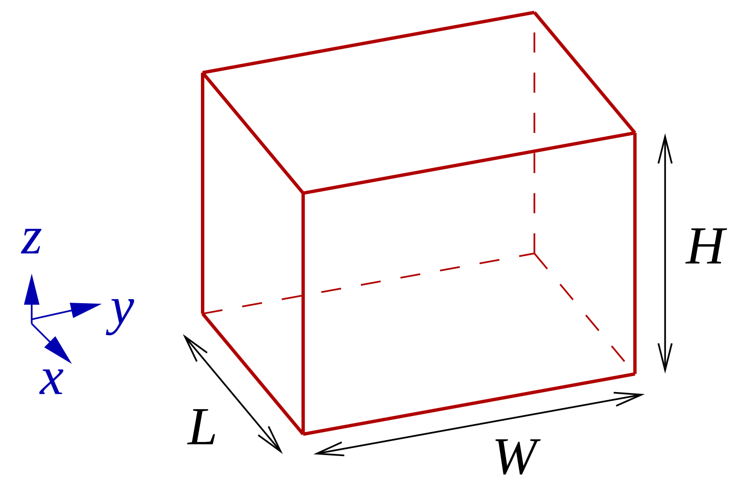
Standing waves in a rectangular cavity

In 3D space, velocity normal to all rigid boundaries is zero:

$$\frac{\partial p}{\partial x} = 0 \qquad \text{for } x = \{0, L\}$$

$$\frac{\partial p}{\partial y} = 0 \qquad \text{for } y = \{0, W\}$$

$$\frac{\partial p}{\partial z} = 0 \qquad \text{for } z = \{0, H\}$$



Complex solution of the 3D wave equation yields

$$\mathbf{p}(t, x, y, z) = X(x)\, Y(y)\, Z(z)\, \mathbf{e}^{j\omega t} \tag{5}$$

which corresponds to a **real** soundfield of the form

$$p(t, x, y, z) = P(x, y, z) \cos \omega t \tag{6}$$

Mode shapes of a cavity

From eq.5, the complex pressure defines the 3D soundfield,

$$\mathbf{p}(t,x,y,z) = X(x)\,Y(y)\,Z(z)\,\mathbf{e}^{j\omega t},$$

whose component mode shapes, $X(x)$, $Y(y)$ and $Z(z)$, each have a **wavenumber** $k = 2\pi/\lambda = \omega/c$ where $\omega = 2\pi f$.

Putting $\mathbf{p}(t,x,y,z)$ into the 3D wave equation gives three equations:

$$\begin{aligned}\left(k_x^2 + \frac{\partial^2}{\partial x^2}\right) X(x) &= 0 \\ \left(k_y^2 + \frac{\partial^2}{\partial y^2}\right) Y(y) &= 0 \\ \left(k_z^2 + \frac{\partial^2}{\partial z^2}\right) Z(z) &= 0 \qquad (7)\end{aligned}$$

where the mode's **resonance frequency** is given by

$$f = \frac{c\,k}{2\pi} = \frac{c}{2\pi}\sqrt{k_x^2 + k_y^2 + k_z^2} \qquad (8)$$

Mode shapes of a cavity

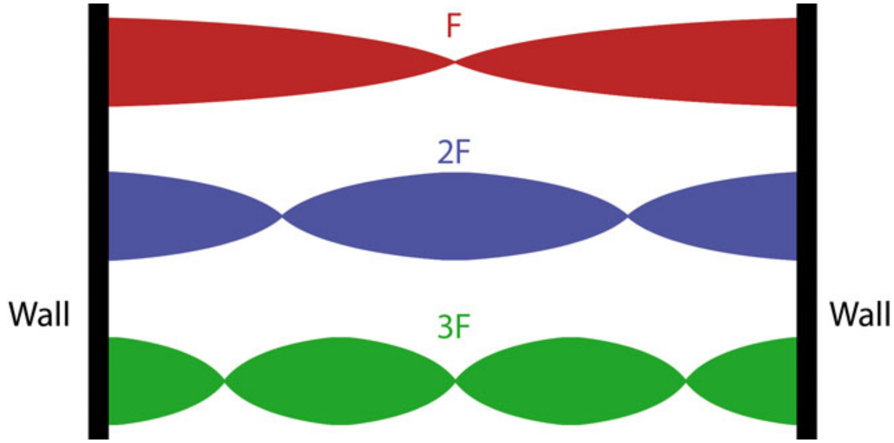
Applying the boundary conditions gives cosine solutions for $X(x)$, $Y(y)$ and $Z(z)$, oscillating sinusoidally over time:

$$\mathbf{p}(x, y, z, t) = a_{lmn} \cos k_x(l)x \; \cos k_y(m)y \; \cos k_z(n)z \; \mathbf{e}^{j\omega t}$$

where $k_x(l) = \frac{l\pi}{L}$, $k_y(m) = \frac{m\pi}{W}$ and $k_z(n) = \frac{n\pi}{H}$.

Resonance frequencies of the modes are quantised by their mode number (l,m,n):

$$f(l, m, n) = \frac{c}{2}\sqrt{\left(\frac{l}{L}\right)^2 + \left(\frac{m}{W}\right)^2 + \left(\frac{n}{H}\right)^2} \qquad (9)$$



Room modes

Axial modes: between two surfaces (strong)

$$f(l) = \frac{c\,l}{2\,X}, \tag{10}$$

mode number (l) = no. $\frac{1}{2}$ wavelengths = no. pressure nodes

Tangential modes: between four surfaces (medium)

$$f(l,m) = \frac{c}{2}\sqrt{\left(\frac{l}{X}\right)^2 + \left(\frac{m}{Y}\right)^2} \tag{11}$$

mode number (l, m) in 2D, for $l, m \in \{1, 2, \ldots\}$

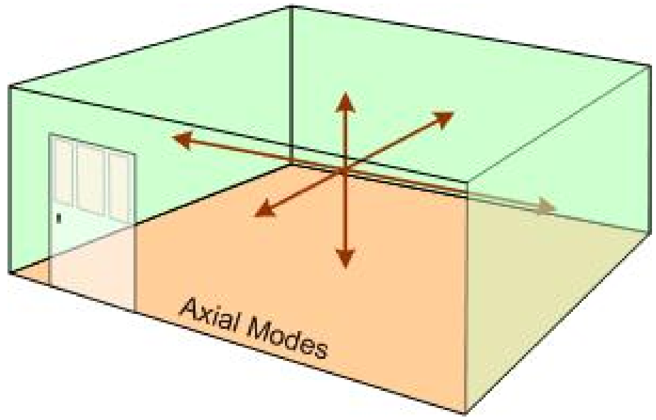
Oblique modes: between six surfaces (weak)

$$f(l,m,n) = \frac{c}{2}\sqrt{\left(\frac{l}{X}\right)^2 + \left(\frac{m}{Y}\right)^2 + \left(\frac{n}{Z}\right)^2} \tag{12}$$

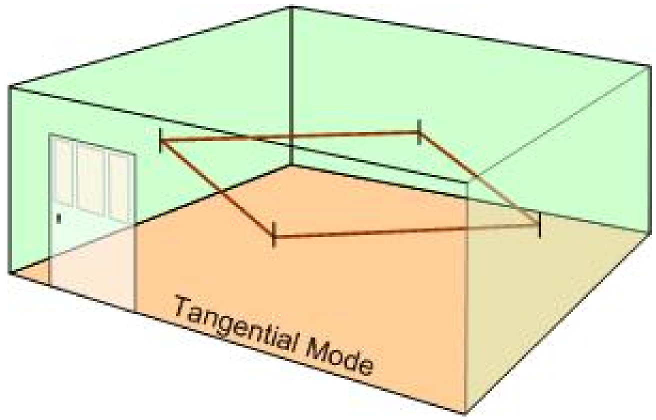
mode number (l, m, n) in 3D, for $l, m, n \in \{1, 2, \ldots\}$

Mode types

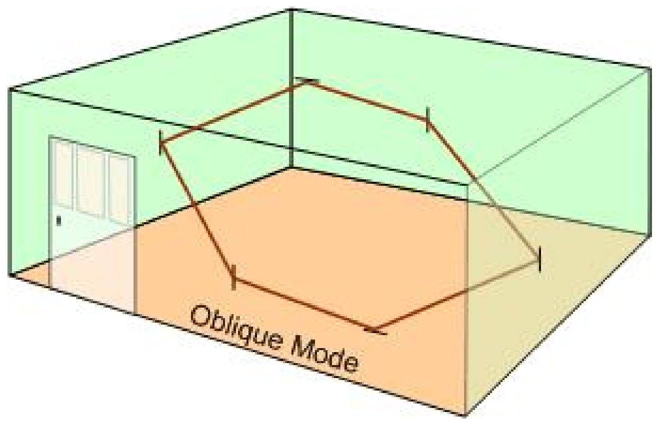
Axial



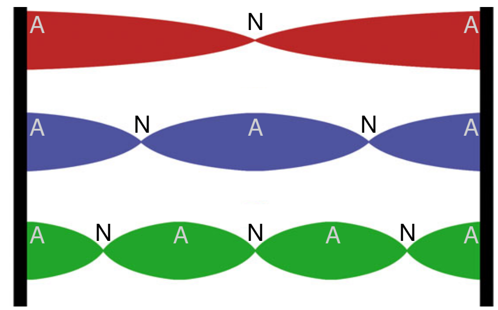
Tangential



Oblique



Nodes and antinodes



Definitions

Node (N): point of no variation
Antinode (A): point of max. variation

Nodal planes

Planes where no pressure variation occurs divide the cavity parallel to the walls. Within each region, pressure varies in phase; adjacent regions are out of phase (antiphase).

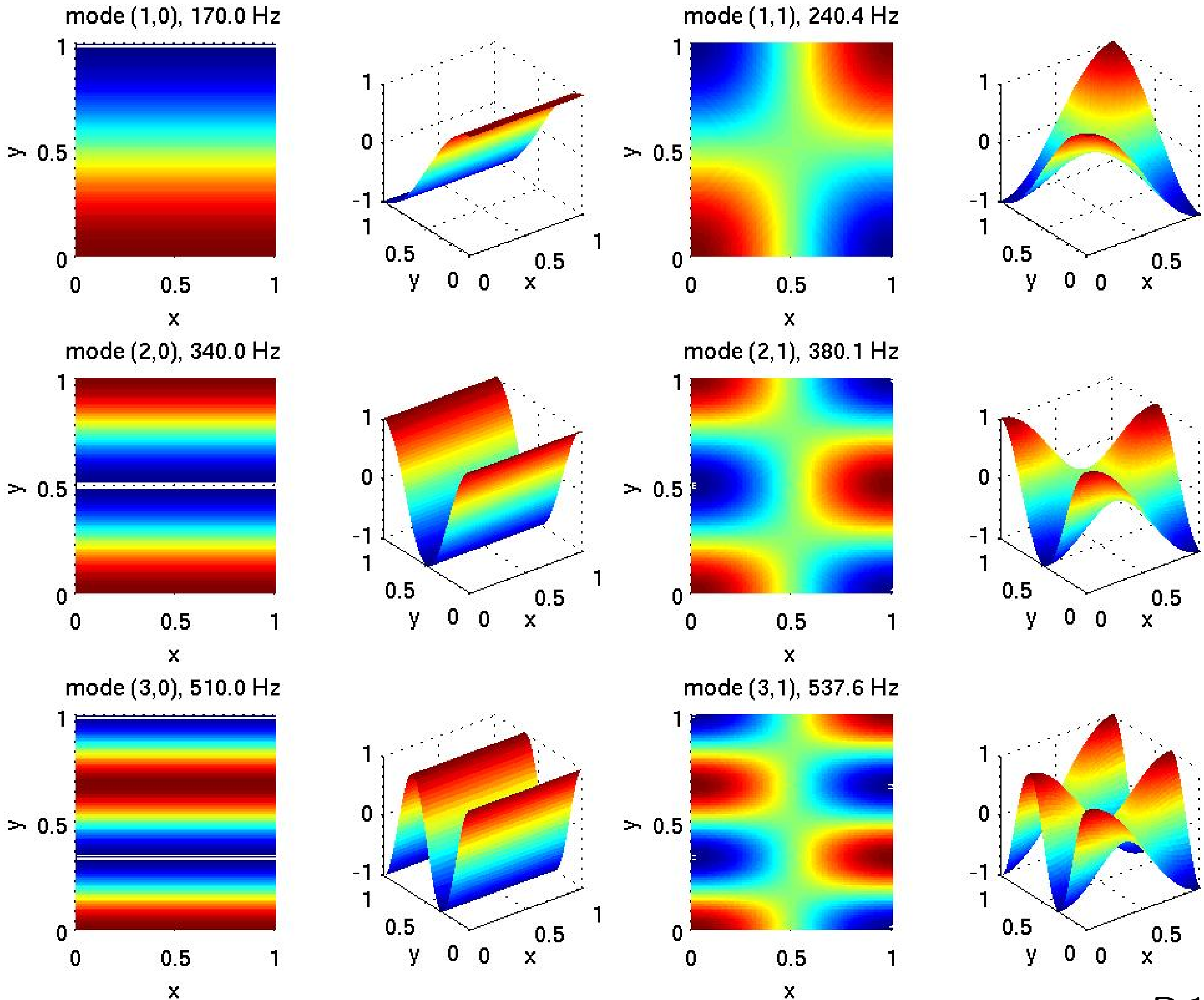
Acoustic behaviour

If source is at a node, it cannot excite the mode; but if it is at an antinode, it gives maximum excitation of that mode.

At a node, microphone reads nothing; at an antinode, it gets maximum reading for that mode.

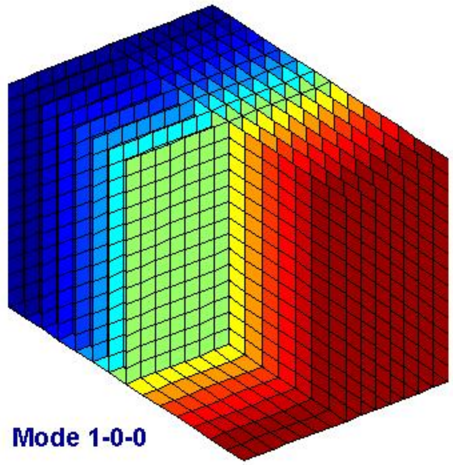
To excite all modes, put source near corner of room; measure with sensor in another (e.g., mike in opposite corner).

Axial modes   Tangential modes

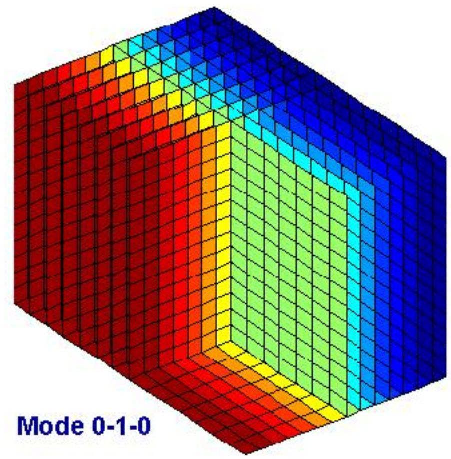


Mode shapes

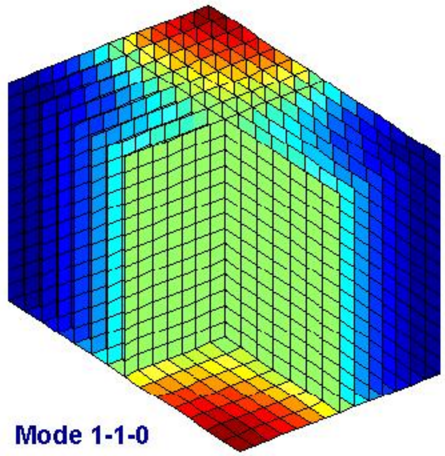
Axial x



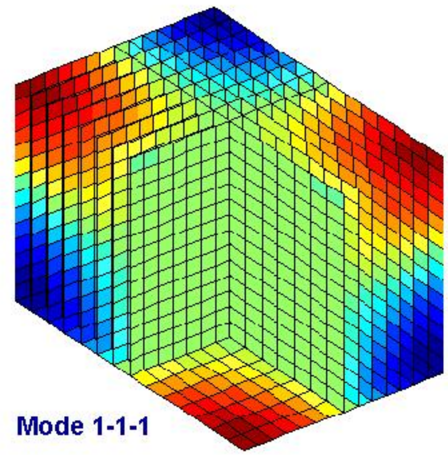
Axial y



Tangential



Oblique



Resonance frequencies of a room

In general, we can express the room mode frequencies

$$\boxed{f(l,m,n) = \frac{c}{2}\sqrt{\left(\frac{l}{X}\right)^2 + \left(\frac{m}{Y}\right)^2 + \left(\frac{n}{Z}\right)^2}} \tag{13}$$

with the mode number (l, m, n) in 3D, for $l, m, n \in \{0, 1, 2, \ldots\}$, which count the number of **nodal planes** along each axis.

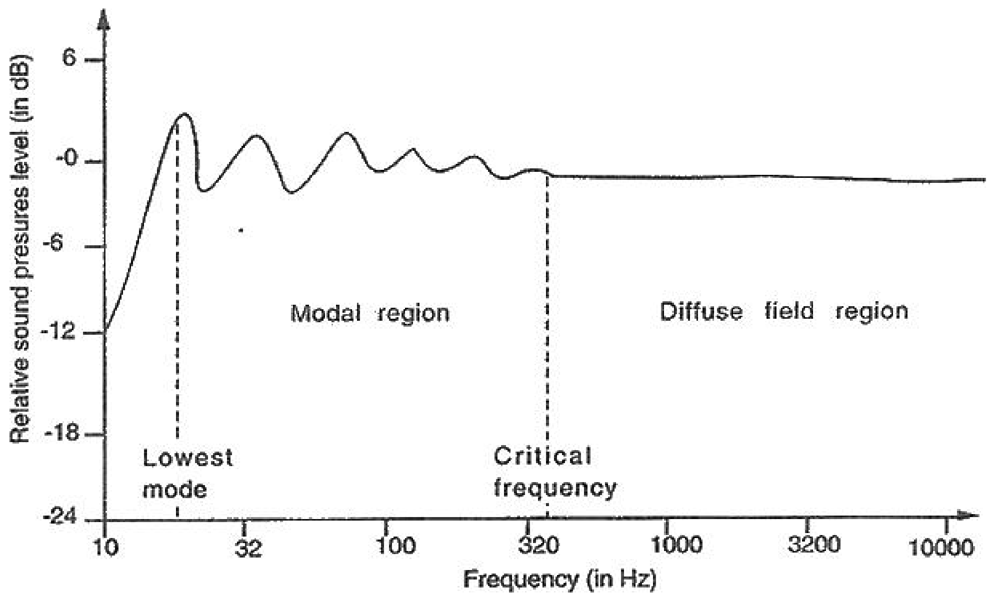
Axial modes have one non-zero dimension and two zeros, e.g., (1-0-0).

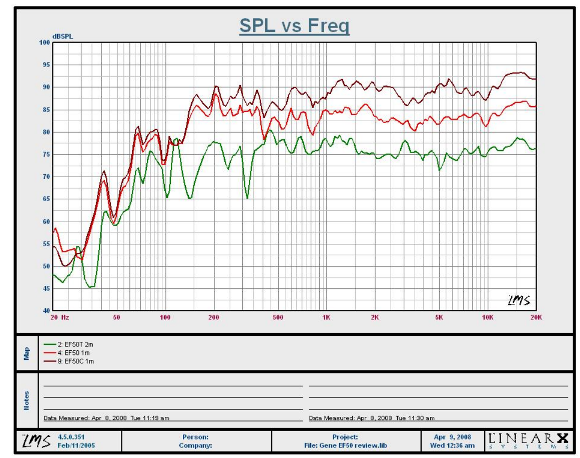
Tangential modes have two non-zero dimensions and one zero, e.g., (1-2-0).

Oblique modes have all three non-zero dimensions, e.g., (1-1-1).

Room frequency response

Room resonance frequencies get closer as frequency rises.





Critical frequency f_{crit} marks when modes start to overlap.

"large" room: f_{crit} below required audio frequency range

"small" room: f_{crit} within required audio frequency range

Cut-off frequency below which there are no resonances to reinforce sound.

Reflected sound waves

- Soundfields in a rectangular cavity
- Standing waves
  - derivation of mode shapes
  - axial, tangential and oblique modes
  - nodes and antinodes
  - resonance frequencies
- Room modes
  - room frequency response
  - critical frequency

Preparation for room acoustics

- What is the absorption coefficient α?
  - find a definition
  - give two examples of materials with different values of α

- How can we characterise the amount of reflection or absorption in a room?
  - find out what is the meaning of *reverberation time*
  - how can it be measured?