EE1.el3 (EEE1023): Electronics III

Acoustics lecture 12
Principles of sound

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Overview for this semester

1. Introduction to sound
2. Human sound perception
3. Noise measurement
4. Sound wave behaviour
5. Reflections and standing waves
6. Room acoustics
7. Resonators and waveguides
8. Musical acoustics
9. Sound localisation

Introduction to sound

- **Definition** of sound
- **Vibration** of matter
  - transverse waves
  - longitudinal waves
- **Sound wave** propagation
  - plane waves
  - pure tones
  - speed of sound

Preparation for Acoustics

- What is a **sound wave**?
  - look up a definition of sound and its properties

- Example of a device
  for manipulating sound
  - write down or
    draw your example
  - explain how it
    modifies the sound

Definitions of sound

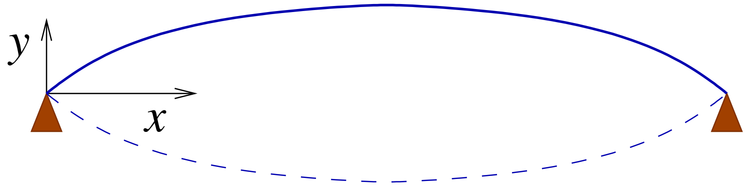
“Sensation caused in the **ear** by the **vibration** of the surrounding **air or other medium**”, Oxford English Dictionary

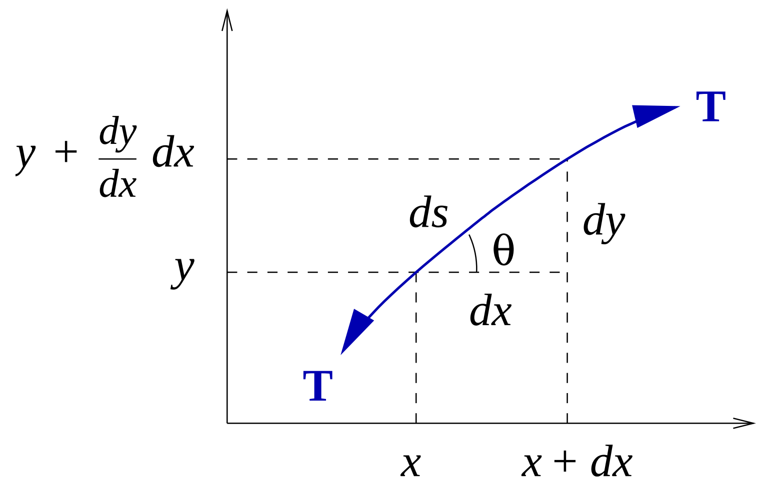
“Disturbances in the **air** caused by **vibrations**, information on which is transmitted to the brain by the sense of **hearing**”, Chambers Pocket Dictionary

“Sound is **vibration** transmitted through a **solid, liquid, or gas**; particularly, sound means those vibrations composed of frequencies capable of being detected by **ears**”, Wikipedia

Vibration of a string

Let us consider forces on an element of the vibrating string with tension T and ρ_L mass per unit length, to derive its equation of motion





Net vertical force from tension between points x and $x + dx$:

$$dF_Y = (T \sin\theta)_{x+dx} - (T \sin\theta)_x \qquad (1)$$

Taylor series expansion

$$f(x+dx) = f(x) + \left(\frac{df}{dx}\right)_x dx + \frac{1}{2}\left(\frac{d^2 f}{dx^2}\right)_x dx^2 + \ldots \qquad (2)$$

Derivation of 1D wave equation

So, we can expand the tension forces on the string element

$$
\begin{aligned}
dF_Y &= \left[(T\sin\theta)_x + \left(\frac{\partial\,(T\sin\theta)_x}{\partial x}\right)_x dx + \ldots\right] - (T\sin\theta)_x \\
&\approx \left(\frac{\partial\,(T\sin\theta)_x}{\partial x}\right)_x dx \qquad (3)
\end{aligned}
$$

For small θ, we assume $\sin\theta \approx \partial y/\partial x$ and constant T, giving

$$dF_Y \approx \frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right)dx \approx T\frac{\partial^2 y}{\partial x^2}dx \qquad (4)$$

Since mass of the element is $\rho_L dx$, Newton gives vertical inertial force

$$dF_Y = \rho_L dx\,\frac{\partial^2 y}{\partial t^2} \qquad (5)$$

Finally, we equate the two vertical forces to obtain

$$\rho_L\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2} \qquad \Rightarrow \qquad \boxed{\frac{\partial^2 y}{\partial t^2} - v^2\frac{\partial^2 y}{\partial x^2} = 0} \qquad (6)$$

where $v = \sqrt{T/\rho_L}$, (cf. lecture 10 on SHM, slide J.5, eq. 3).

Solutions to 1D wave equation

Suppose a simple solution to the 1D wave equation, eq. 6:

$$y(t,x) = \cos\left(\omega\left(t - \frac{x}{v}\right)\right) \tag{7}$$

Taking derivatives w.r.t. time and position gives

$$\frac{\partial y}{\partial t} = -\omega \sin\left(\omega\left(t - \frac{x}{v}\right)\right) \qquad \frac{\partial y}{\partial x} = \frac{\omega}{v} \sin\left(\omega\left(t - \frac{x}{v}\right)\right)$$

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 \cos\left(\omega\left(t - \frac{x}{v}\right)\right) \qquad \frac{\partial^2 y}{\partial x^2} = -\frac{\omega^2}{v^2} \cos\left(\omega\left(t - \frac{x}{v}\right)\right)$$

Finally, we can confirm it satisfies the wave equation:

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial x^2} = 0$$

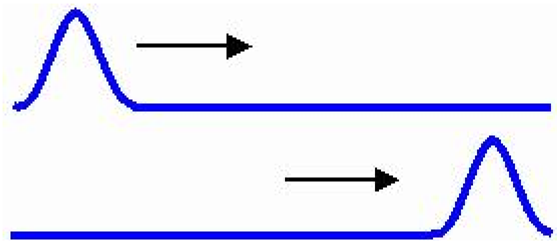
More generally, solutions exist of the form

$$y(t,x) = g\left(t - \frac{x}{v}\right) + h\left(t + \frac{x}{v}\right) \tag{8}$$

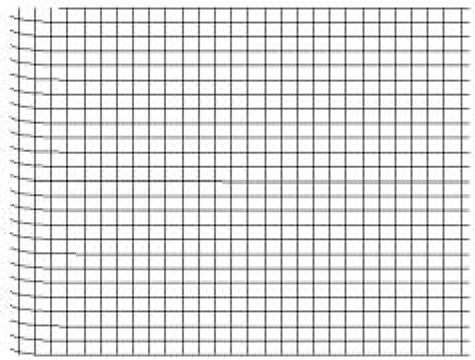
where $g(\cdot)$ and $h(\cdot)$ can be arbitrary waveforms.

Transverse waves

Transverse vibrations are perturbations *across* the direction of wave travel:
e.g., a pulse on a string



$$y(t,x) = g\left(t - \frac{x}{v}\right)$$



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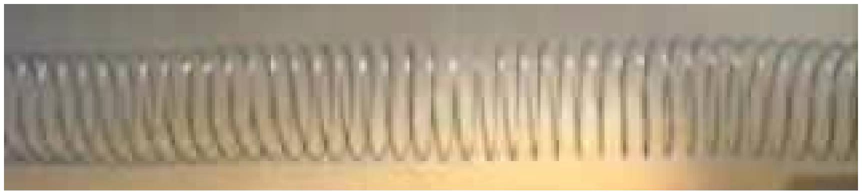
What is a sound wave?

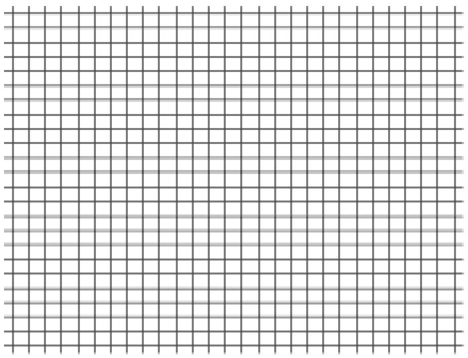
- small perturbation of particles
- adiabatic process (i.e., fast) without loss of entropy
- propagates (travels) longitudinally in a medium
- pressure and velocity fluctuations can be measured
- sound perceived in humans by ears and cochlea

Longitudinal waves

longitudinal vibrations are perturbations *along* the direction of wave travel:

e.g., a pulse along a spring





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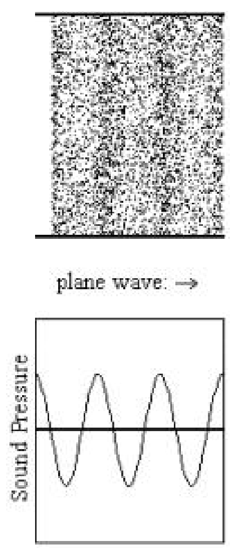
Sound propagating in 1D (plane waves)

Consider a **planar source** (e.g. piston) that emits sound in a normal direction:

- infinite source of pressure or velocity fluctuation
- uniform continuous medium without boundaries to the direction of wave propagation
  e.g., in pipes, vocal tract, exhaust, ducting, far field
- plane wave propagation of the wavefronts



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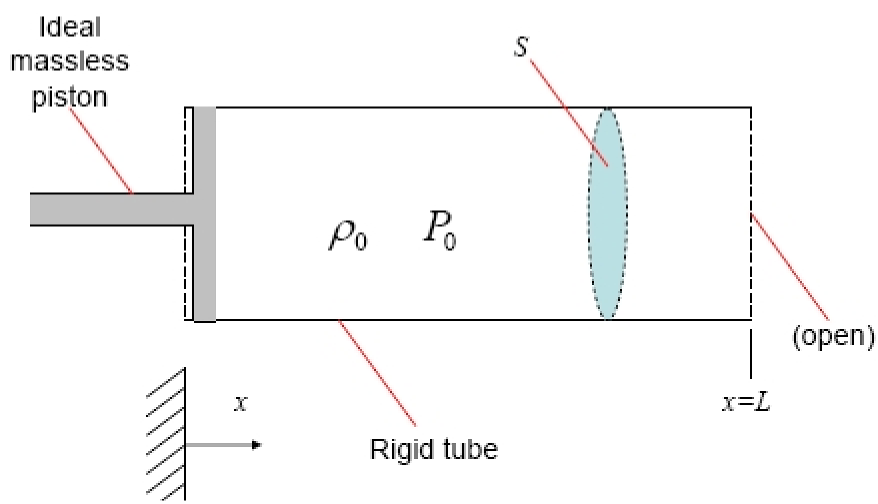
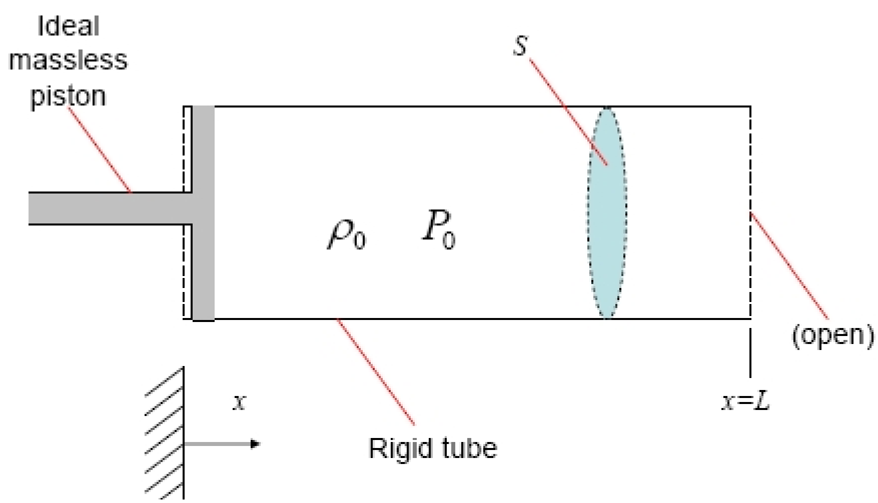


Interpretation of wave propagation

Suppose a piston delivers an impulse to the air in a tube:



1. Piston pushes in, squashing the air in contact
2. As density rises, air pressure increases
3. Pressure difference accelerates the adjacent air
4. Movement of air particles releases the pressure at the piston, and squashes the next layer of air
5. And so the process continues...

1D sound wave equation

Assuming a perfect gas medium, pressure $P = P_0 + p$ of a given volume depends on density ρ, specific gas constant $r = R/M$, and absolute temperature T gives: $P = \rho r T$
Adiabatic wave propagation states

$$\left(\frac{P_0 + p}{P_0}\right) = \frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^{\gamma} \tag{9}$$

where γ is the ratio of heat capacities for the medium.

Considering the continuity of matter and inertial forces, we can obtain the 1D plane wave equation (Kinsler et al., 2000):

$$\frac{\partial^2 p}{\partial t^2} = c^2 \frac{\partial^2 p}{\partial x^2} \quad \Rightarrow \quad \boxed{\frac{\partial^2 p}{\partial t^2} - c^2 \frac{\partial^2 p}{\partial x^2} = 0} \tag{10}$$

$$\text{where} \quad \boxed{c = \sqrt{\gamma r T}} \tag{11}$$

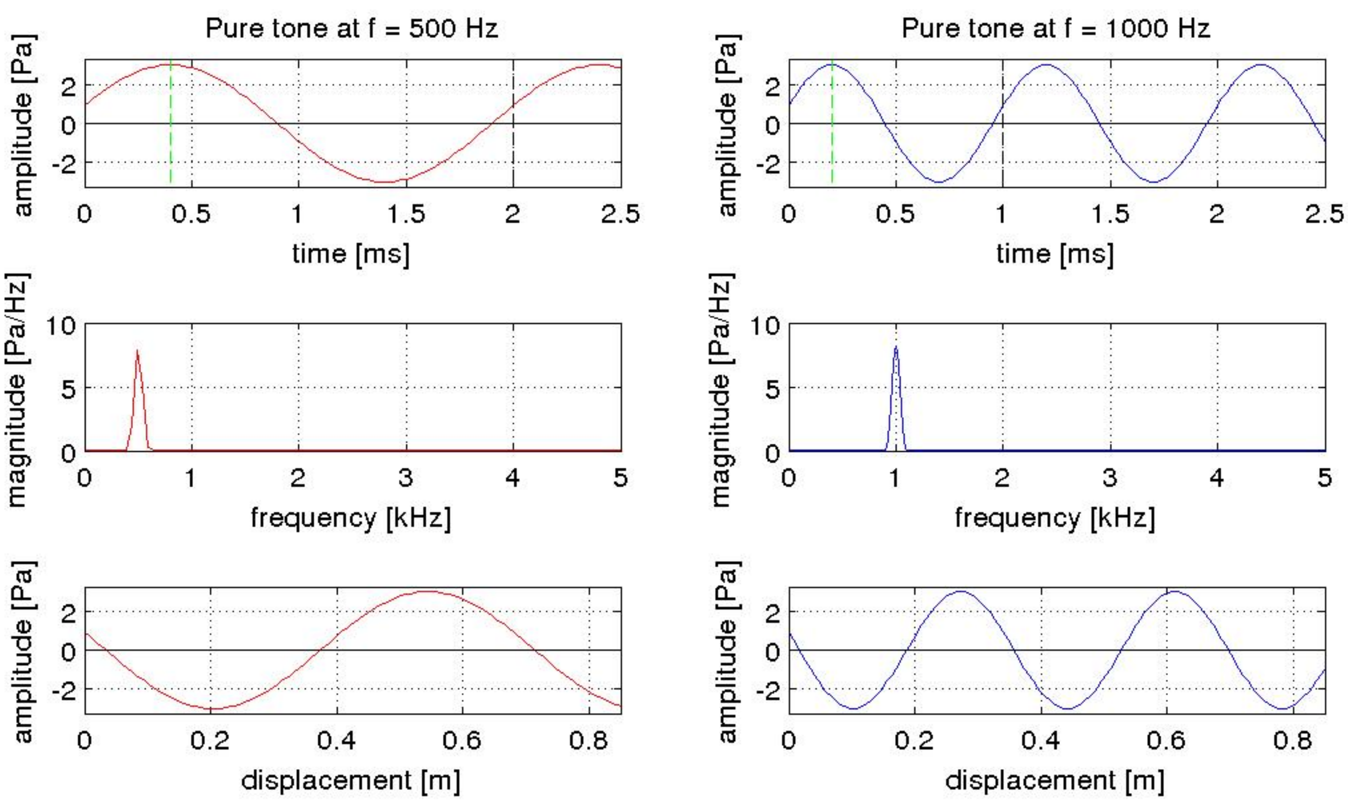
[L.E. Kinsler et al., *Fundamentals of acoustics*, 4th ed., New York: Wiley, 2000.]
see pp. 114-121 (shelfmark 534/KIN)

Soundfield of a pure tone

As before, one solution to the 1D sound wave equation is the soundfield produced by a sinusoidal plane wave:

$$p(t, x) = a \cos\left(\omega \left(t - x/c\right) + \phi\right),$$

where peak amplitude is a, frequency $f = 1/\tau$ reciprocal of the period τ, angular frequency is $\omega = 2\pi f$, sound speed c, phase offset ϕ, and wavelength $\lambda = c/f$.



Speed of sound, c

For a 3D soundfield $p(t, \mathbf{x}) = f(t - (\mathbf{x}.\mathbf{e}/c))$, the plane wave travels along unit vector $\mathbf{e}$ at the speed of sound $c = \sqrt{\gamma r T}$.

Sound speed depends on the inertia and stiffness of the medium, which are functions of ambient temperature and pressure (cf. natural frequency of SHM). In air, eq. 11 gives

$$\begin{aligned} c_{\text{air},0^\circ\text{C}} &= \sqrt{1.4 \times 287 \times 273} \approx 331\,\text{m}\,\text{s}^{-1} \\ c_{\text{air},20^\circ\text{C}} &= \sqrt{1.4 \times 287 \times 293} \approx 343\,\text{m}\,\text{s}^{-1}. \end{aligned}$$

For calculations at room temperature, the speed of sound is usually taken as $c_{\text{air}} = 340\,\text{m}\,\text{s}^{-1}$.

Sound speed in helium, $c_{\text{He}} = 972\,\text{m}\,\text{s}^{-1}$

Sound speed in water, $c_{\text{H}_2\text{O}} = 1500\,\text{m}\,\text{s}^{-1}$

Introduction to acoustics

- **Definition of sound**
  - pressure perturbation in a medium
  - propagates isentropically
- **Derivation of 1D wave equation**
  - transverse and longitudinal vibrations
- **Plane-wave sound propagation**
  - sound pressure
  - harmonic solution (pure tone, sine wave)
  - speed of sound

Preparation for Human hearing

- Physiology of the ear and cochlea
  - sketch all parts of the ear that are used for auditory perception

- Find out how you would do an experiment to measure:
  - loudness,
  - pitch, or
  - critical bands