EE1.el3 (EEE1023): Electronics III

Mechanics lecture 8
Kinematics, motion of a rigid body

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Kinematics (2)

- **motion of a rigid body**
- relative motion
- rotational motion
- general motion in a cartesian reference frame

What do you know about rotational motion?

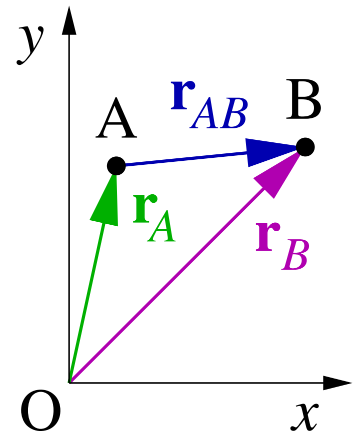
- how do you define angular velocity?
- how can you calculate angular velocity?
- as a vector, what does its direction mean?
- how does angular velocity relate to angular displacement and acceleration?

Motion of a rigid body

Relative motion

Relative displacement

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A \qquad (1)$$



Relative velocity

$$\frac{d}{dt}(\mathbf{r}_{AB}) = \frac{d}{dt}(\mathbf{r}_B) - \frac{d}{dt}(\mathbf{r}_A)$$

$$\Rightarrow \qquad \dot{\mathbf{r}}_{AB} = \dot{\mathbf{r}}_B - \dot{\mathbf{r}}_A \qquad (2)$$

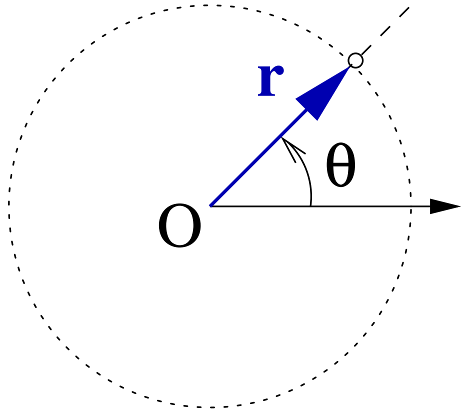
Relative acceleration

$$\frac{d^2}{dt^2}(\mathbf{r}_{AB}) = \frac{d^2}{dt^2}(\mathbf{r}_B) - \frac{d^2}{dt^2}(\mathbf{r}_A)$$

$$\Rightarrow \qquad \ddot{\mathbf{r}}_{AB} = \ddot{\mathbf{r}}_B - \ddot{\mathbf{r}}_A \qquad (3)$$

Angular motion

Angular velocity is the rate of rotation in radians:



$$\omega \;=\; \dot{\theta} \;=\; \frac{d\theta}{dt} \qquad (4)$$

averaged over one period, we get

$$\bar{\omega} \;=\; \frac{\int_0^{2\pi} d\theta}{\int_0^T dt} \;=\; \frac{2\pi}{T}$$

In the plane of rotation, angular velocity has magnitude

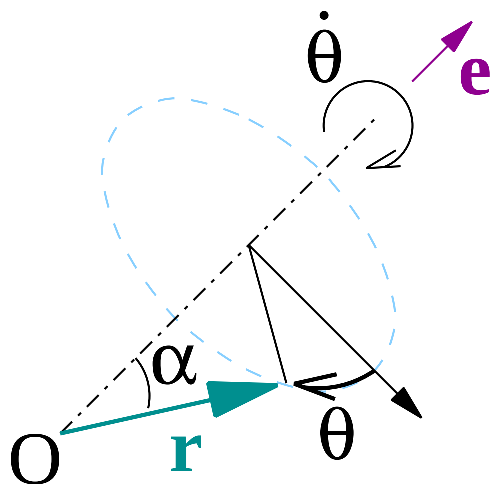
$$\omega \;=\; \frac{\dot{r}}{r} \qquad (5)$$

where $\dot{r}$ is speed and r is radial distance from axis.

Angular velocity can also be expressed in vector form:

$$\boldsymbol{\omega} \;=\; \frac{d\boldsymbol{\theta}}{dt} \;=\; \frac{\mathbf{r} \times \dot{\mathbf{r}}}{r^2} \qquad (6)$$

Properties of angular velocity



Angular velocity is a vector:

$$\boldsymbol{\omega} \;=\; \dot{\boldsymbol{\theta}} \;=\; \dot{\theta}\,\mathbf{e}$$

$$= \begin{pmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix} \qquad (7)$$

with its direction represented by a unit vector $\mathbf{e}$ $\parallel$ to the axis of rotation, as per RH rule

- Magnitude

$$\omega \;=\; |\boldsymbol{\omega}| \;=\; \sqrt{\omega_X{}^2 + \omega_Y{}^2 + \omega_Z{}^2} \qquad (8)$$

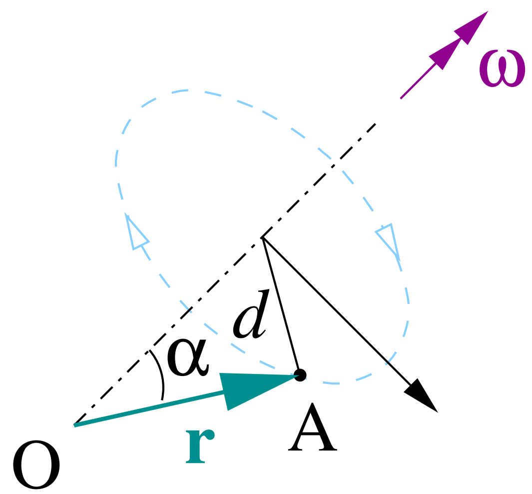
- Direction

$$\mathbf{e} \;=\; \frac{\boldsymbol{\omega}}{|\boldsymbol{\omega}|} \;=\; \frac{1}{\omega}\begin{pmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{pmatrix} \qquad (9)$$

Circular motion

Point A follows a circular path with tangential speed $\dot{r} = \omega d$ in the plane, and

$$d = r \sin\alpha \qquad (10)$$



so that

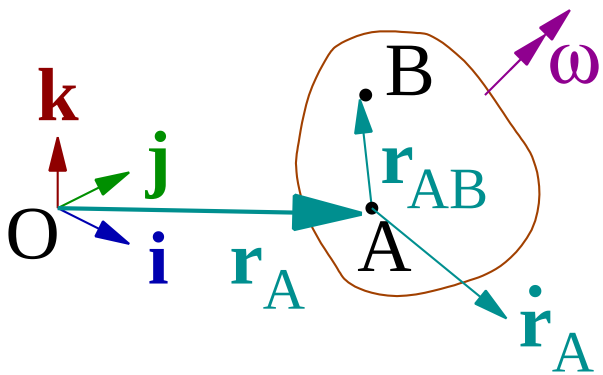
$$\dot{r} = \omega r \sin\alpha \qquad (11)$$

At any instant, its (linear) velocity is

$$\dot{\mathbf{r}} \;=\; \boldsymbol{\omega} \times \mathbf{r} \qquad (12)$$

General motion

If points A and B are a fixed distance apart,



$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{AB} \quad (13)$$

and

$$\dot{\mathbf{r}}_B = \dot{\mathbf{r}}_A + \dot{\mathbf{r}}_{AB} \quad (14)$$

From the body's angular velocity, we know

$$\dot{\mathbf{r}}_{AB} = \boldsymbol{\omega} \times \mathbf{r}_{AB} \quad (15)$$

Putting this into (14), we get the equation of motion for B

$$\dot{\mathbf{r}}_B = \dot{\mathbf{r}}_A + \boldsymbol{\omega} \times \mathbf{r}_{AB} \quad (16)$$

simply expressed in terms of fixed directions $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$

Summary of Kinematics (2)

- Motion of a rigid body
  - relative motion

- Rotational motion
  - angular velocity
  - circular motion
  - general motion in a fixed reference frame

Preparation for Rigid body dynamics

- What is **angular momentum**?
  - look up the equation for it
  - define all the terms in the equation

- What is **angular inertia**?
  - look up its definition
  - give one example