Quiz!

Total score out of 20 marks

Question 1

The equation for kinetic energy is $E_{\text{KE}} = \frac{1}{2}mv^2$. Using SI base units, what is an equivalent way to express 100 J?

$\text{dim}(m) = \text{M}$ and $\text{dim}(v) = \text{L T}^{-1}$,

So, $\text{dim}(E_{\text{KE}}) = \text{M L}^2 \text{ T}^{-2}$

$\Rightarrow$ $\boxed{\mathbf{100\,J = 100\ kg\ m^2\ s^{-2}}}$

Question 2

The frequency of a note played on a guitar string of length L is $f = \frac{1}{2L}\left(\frac{T}{\mu}\right)^a$ where the tension T has dimension $\mathsf{M\,L\,T}^{-2}$ and linear mass μ has $\mathsf{M\,L}^{-1}$. What is the value of a by dimensional analysis?

$\dim(L) = \mathsf{L}$ and $\dim(f) = \mathsf{T}^{-1}$,

$\dim\left(\frac{T}{\mu}\right) = \mathsf{L}^2\ \mathsf{T}^{-2}$

So, $\mathsf{T}^{-1} = \mathsf{L}^{-1}\left(\mathsf{L}^2\mathsf{T}^{-2}\right)^a$

Hence, considering dimensions in either L or T, we get

$\Rightarrow \boxed{a = \frac{1}{2}}$

Question 3

Hubble's constant, which describes the rate of expansion of the universe, is usually quoted as 71 (km/s)/Mpc. What is the SI unit for this constant?
(Hint: $1\,\text{Mpc} = 10^6\,\text{parsec} = 30.9 \times 10^{15}\,\text{m}$.)

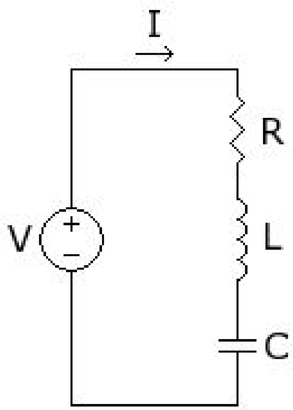
Dimensions of the units:
$\dim((\text{km/s})/\text{Mpc}) = (\text{L T}^{-1})/(\text{L}) = \text{T}^{-1}$

$\Rightarrow$ SI base unit is $\boxed{\mathbf{s}^{-1}}$

An alternative SI unit with same dimensions is hertz, Hz.

Question 4

In a series RLC tuning circuit, the Q factor of the resonance is a dimensionless quantity that describes the strength or 'quality' of tuning, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$.
For a given application, you want to double the resistance in relation to your existing design, but keep the same inductor. What change must you make to the capacitance to have the same Q factor?



$$\frac{1}{R_1}\sqrt{\frac{L}{C_1}} = \frac{1}{R_2}\sqrt{\frac{L}{C_2}}$$

$$\left(\frac{R_2}{R_1}\right)^2 = \frac{C_1}{C_2} = 4$$

$$\Rightarrow \quad C_2 = \frac{C_1}{4}$$

Question 5

Given two forces, $\mathbf{F}_1 = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{F}_2 = -\mathbf{i} + 2\mathbf{j}$, find the magnitude of $|\mathbf{F}_1 - \mathbf{F}_2|$.

$$(\mathbf{F}_1 - \mathbf{F}_2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$|\mathbf{F}_1 - \mathbf{F}_2| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Question 6

If there is a force $\mathbf{N} = 5\mathbf{j}$ and we define $\mathbf{e}$ to be a unit vector in the direction of $\mathbf{F}_1$ from the previous question, what is the size of $\mathbf{N}$'s component along $\mathbf{e}$?

$$\mathbf{e} = \frac{1}{\sqrt{10}} \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\mathbf{N}.\mathbf{e} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix} \frac{1}{\sqrt{10}}$$

$$= \frac{15}{\sqrt{10}}$$

Question 7

A raindrop reaches its terminal velocity of $5.4\,\text{ms}^{-1}$ when the drag force from air resistance reaches $9.8 \times 10^{-6}\,\text{N}$. What is the mass of the raindrop?

At terminal velocity, the raindrop is in dynamic equilibrium, so the drag force from air resistance is equal to the weight:

$$W = m\,g$$

$$m = \frac{9.8 \times 10^{-6}}{9.8}$$

$$= 10^{-6}\,\text{kg} = 1\,\text{mg}$$

Question 8

A weight $\mathbf{W} = 150(-\mathbf{j})$ N is added at the top of a ladder at position $(2, 3)$. Considering moments about the foot of the ladder at the origin $(0, 0)$, what is the increase in reaction force $\mathbf{R}$ in the direction $-\mathbf{i}$ out from the wall that is needed to keep the ladder still?

The moments about the origin must be equal and opposite to maintain equilibrium:

$$
\begin{aligned}
2 \times 150 &= 3R \\
\Rightarrow \quad \mathbf{R} &= 100(-\mathbf{i})\,\text{N}
\end{aligned}
$$

Question 9

A particle has a displacement $\mathbf{r}(t) = \frac{3}{2}t^2\,\mathbf{i} + \cos(10\pi t)\,\mathbf{j}$ as it oscillates in an electric field. What is its velocity $\dot{\mathbf{r}}(t)$ at time $t = 50\,\text{ms}$?

$$
\begin{aligned}
\dot{\mathbf{r}}(t) &= 3t\,\mathbf{i} - 10\pi\sin(10\pi t)\,\mathbf{j} \\
\Rightarrow \quad \dot{\mathbf{r}}(0.05) &= \frac{3}{20}\,\mathbf{i} - 10\pi\,\mathbf{j}
\end{aligned}
$$

Question 10

At 30 mph, the angular velocity of a wheel on a car is $50\mathbf{j}\,\mathrm{rad\,s^{-1}}$. As the car starts to turn left around a sharp corner, it has an angular velocity of $1\mathbf{k}\,\mathrm{rad\,s^{-1}}$. What is the total angular velocity of the wheel now?

$$\boxed{\boldsymbol{\omega} = \begin{pmatrix} 0 \\ 50 \\ 1 \end{pmatrix} \mathrm{rad\,s^{-1}}}$$