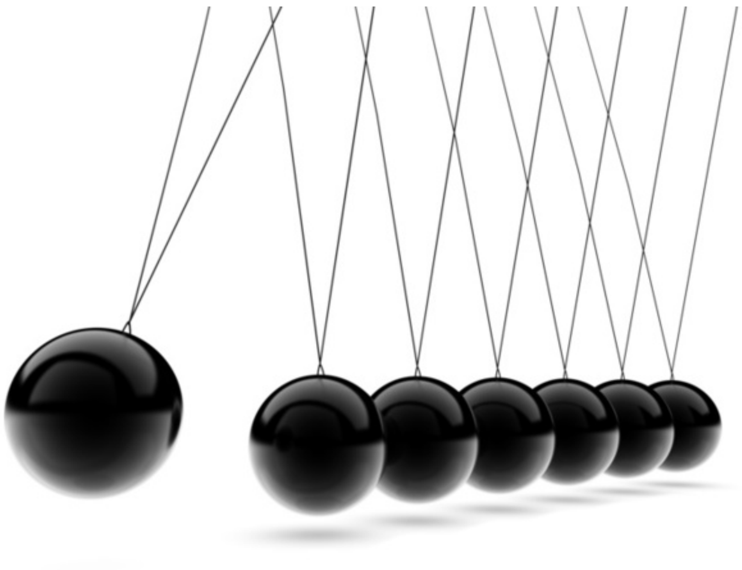
EE1.el3 (EEE1023): Electronics III

Mechanics lecture 6

Dynamics, conservation of energy

Dr Philip Jackson



www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3

UNIVERSITY OF SURREY

Dynamics (2)

- Motion in a force field
- Conservation of energy
- Potential energy

Preparation for Dynamics (2)

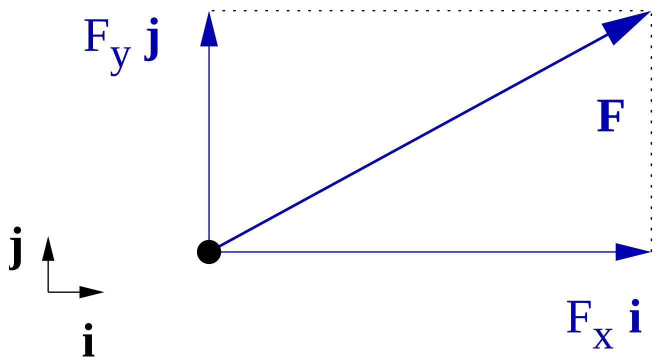
- **Conservation of energy**

- In mechanics, what is a conservative force?

- Conservative forces
  - *find two examples of a conservative force*

- Non-conservative forces
  - *find two examples of a non-conservative force*

Conservative forces



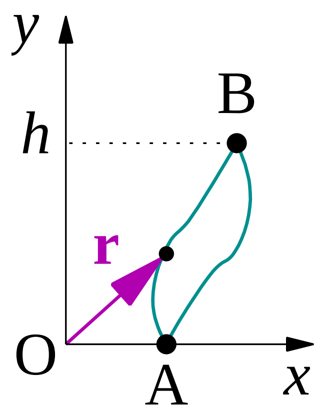
Given a force,

$$\begin{aligned} \mathbf{F} &= F_x\,\mathbf{i} + F_y\,\mathbf{j} \\ &= \begin{pmatrix} F_x \\ F_y \end{pmatrix} \end{aligned} \tag{1}$$

we say it is **conservative** if the work done by the force around a closed path is zero:

$$W = \oint \mathbf{F}.\mathbf{dr} = 0 \tag{2}$$

Motion in a gravitational force field



Work done by gravity alone:

$$\begin{aligned} W_{AB} &= -mgh \\ W_{BA} &= +mgh \end{aligned} \quad (3)$$

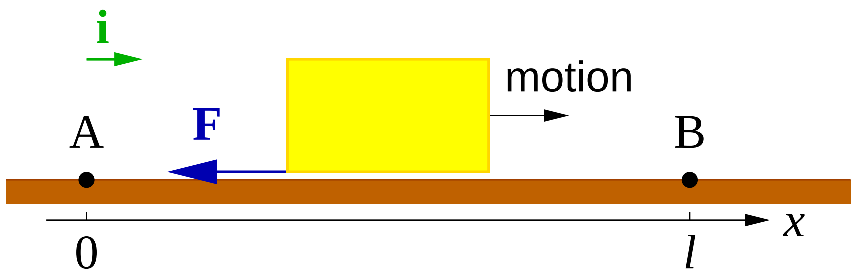
Considering the total work done around the closed path from A to B and back to A, we get

$$\oint_{ABA} dW = W_{AB} + W_{BA} = 0 \quad (4)$$

Therefore, gravitational force is *conservative*
— what you put in is what you get out!

Frictional force

The frictional force always opposes the motion.



$$W_{AB} = (-F\,\mathbf{i}).(l\,\mathbf{i}) = -Fl$$
$$W_{BA} = (F\,\mathbf{i}).(-l\,\mathbf{i}) = -Fl \quad (5)$$

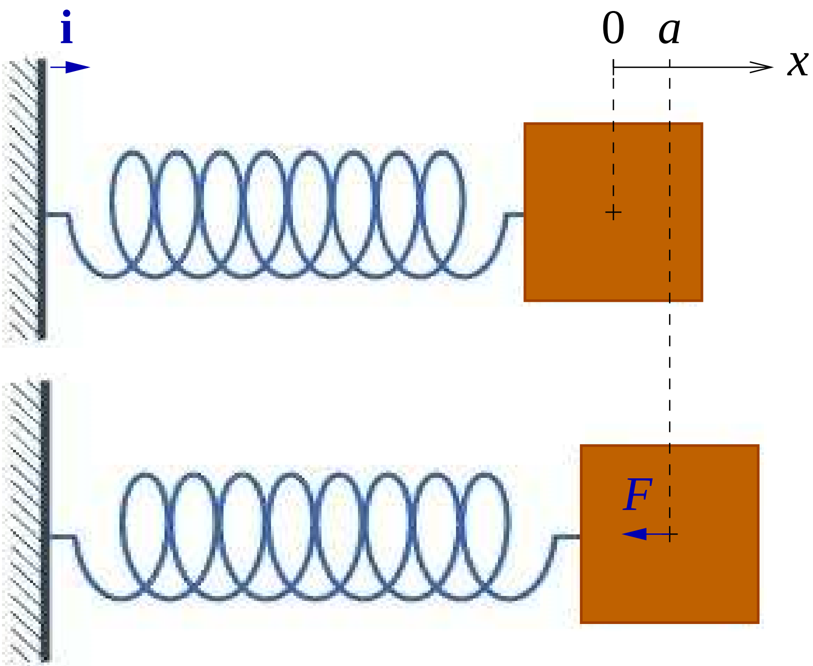
Combining these to complete the loop, we obtain

$$\oint_{ABA} dW = W_{AB} + W_{BA} = -2Fl \neq 0 \quad (6)$$

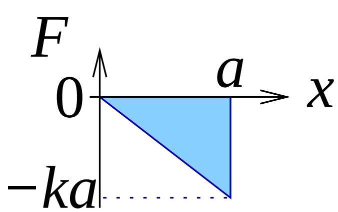
so friction is not a conservative force — it is **lossy**!

Elastic force of a spring

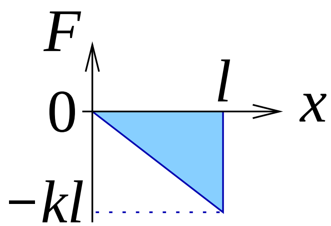
Work done is the integral of force over displacement:



$$
\begin{aligned}
W &= \int_0^a \mathbf{F.dx} \\
&= \int_0^a (-kx\,\mathbf{i}).(dx\,\mathbf{i}) \\
&= \int_0^a (-kx)\,dx \\
&= -k\left[\frac{1}{2}x^2\right]_0^a \\
&= \frac{-ka^2}{2} \qquad (7)
\end{aligned}
$$



Elastic force



Work done by the spring:

$$W_{AB} = \left[-\frac{kx^2}{2}\right]_A^B = -\frac{kl^2}{2}$$

$$W_{BA} = \left[-\frac{kx^2}{2}\right]_B^A = +\frac{kl^2}{2} \qquad (8)$$

Considering the closed path from A to B to A, we obtain

$$\oint_{ABA} dW = -\frac{kl^2}{2} + \frac{kl^2}{2} = 0 \qquad (9)$$

so the elastic force is *conservative*.

Conservative and non-conservative forces

Conservative forces

- Inertial
  $\mathbf{F} = m\ddot{\mathbf{r}}$

- Elastic
  $\mathbf{F} = -k\mathbf{x}$

- Gravitational
  $\mathbf{F} = -mg\mathbf{j}$

- Electrostatic
  $\mathbf{F} = \mathbf{E}q$

- Magnetic
  $\mathbf{F} = q\dot{\mathbf{r}} \times \mathbf{B}$

Lossy forces

- Friction
  $F = \mu N$

- Drag
  $F = \frac{1}{2}\rho v^2 A C_D$

- Damping
  $\mathbf{F} = -b\dot{\mathbf{x}}$

Potential energy

Potential energy is work done **against** a conservative force. It is the complement (i.e., negative) of the work done **by** the force as a body moves from A to B:

$$U_{AB} \;=\; -\int_A^B \mathbf{F}.\mathbf{dr} \tag{10}$$

If no work is done, the two points have equal potential.

Conservation of energy

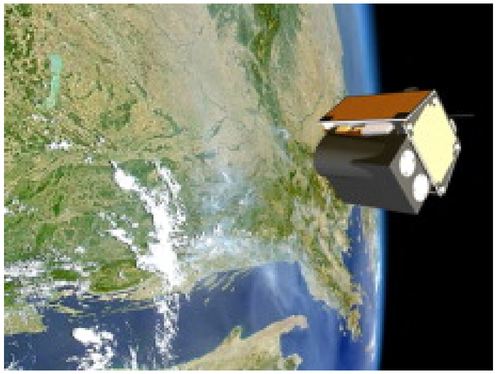
When only conservative forces act, energy is conserved:

$$W_{AB} + U_{AB} \;=\; 0 \tag{11}$$

So, we can treat motion under these forces as exchange from one form of energy to another, without loss. E.g., work vs. gravity $W = -mgh$, gravitational potential $U = mgh$.

Summary of Dynamics (2)

- Work done in a force field
  - conservative forces
  - lossy forces
- Conservation of energy
  - potential energy



Preparation for Moments of a force

- **Motion of a rigid body**

- What is the moment of a force?
  - *give the equation for a moment*
- What is an example of torque?
  - *find a definition of torque*
- What does direction of torque mean?
  - *illustrate with an example*