EE1.el3 (EEE1023): Electronics III

Mechanics lecture 5
Dynamics, forces in motion

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www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3

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Dynamics (1)

- **Introduction to dynamics**

- Momentum
- Work and power
- Energy
- Examples

Ross Land / AP

Preparation for Dynamics (1)

- Work through the exercises given on the web site:

  www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3 > Exercises

- **Forces in motion**

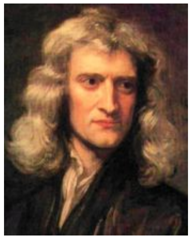
- Newton's laws of motion

  - what are they?

- Illustrate the laws

  - draw three sketches to illustrate each law

Newtons laws of motion



1. **Inertia:** a body stays at rest unless acted upon by an external force; a body in motion travels with constant velocity unless acted upon by an external force

2. **Change:** a force acting on a body causes acceleration with magnitude inversely proportional to the body's mass in the direction of force

$$\ddot{\mathbf{r}} = \frac{\mathbf{F}}{m}$$

3. **Opposition:** forces occur in equal and opposite pairs, if A exerts force on B then B exerts force on A:

$$\mathbf{R} = -\mathbf{F}$$

Momentum

- Linear momentum is defined as

$$\mathbf{p} = m\,\dot{\mathbf{r}} \tag{1}$$

which gives Newton's 2nd law as

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d\,(m\dot{\mathbf{r}})}{dt} \tag{2}$$

- For constant mass $dm/dt = 0$, and so we have

$$\mathbf{F} = m\frac{d\dot{\mathbf{r}}}{dt} = m\,\ddot{\mathbf{r}} \tag{3}$$

Momentum equation

With constant mass, force causes a change in momentum through acceleration

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d\left(m\dot{\mathbf{r}}\right)}{dt} = m\ddot{\mathbf{r}}. \qquad (4)$$

Integrating over time gives the net momentum change

$$\begin{aligned}\int_{t_1}^{t_2} \mathbf{F}\, dt &= \int_{t_1}^{t_2} m\ddot{\mathbf{r}}\, dt = \left[m\dot{\mathbf{r}}\right]_{t_1}^{t_2} \\ &= m\dot{\mathbf{r}}_2 - m\dot{\mathbf{r}}_1 = \mathbf{p}_2 - \mathbf{p}_1 \qquad (5)\end{aligned}$$

Note: if no resultant force acts, *momentum is* **conserved**.

Momentum example

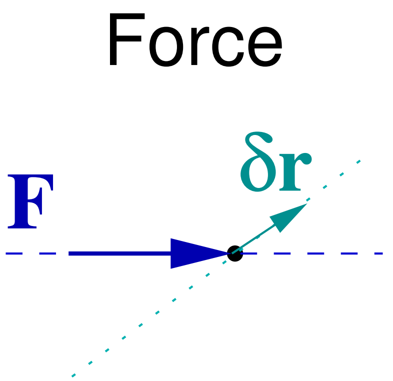
What is the linear momentum of Dave on his bike, $m = 120\,\text{kg}$, at speed $v = 4.4\,\text{m}\,\text{s}^{-1}$?

[Electronics III exam paper 2009]

Work

The **work** done is equal to:

1. the force times the distance along its line of application
2. the parallel force component times the displacement



Linear work is $W_F = \mathbf{F}.\boldsymbol{\delta}\mathbf{r}$

Power

Power is the rate of work

Average power over a short time interval δt is

$$P_F = \mathbf{F}.\frac{\boldsymbol{\delta}\mathbf{r}}{\delta t}$$

Instantaneous power is the limit as $\delta t \to 0$:

$$\begin{aligned} P_F &= \mathbf{F}.\frac{d\mathbf{r}}{dt} \\ &= \mathbf{F}.\dot{\mathbf{r}} \qquad (6) \end{aligned}$$

Energy equation

For a particle, which does not support any rotational component, we have simple expressions for work

$$W = W_F = \mathbf{F}.\boldsymbol{\delta}\mathbf{r} \qquad (7)$$

and for power, the rate of working

$$P = P_F = \mathbf{F}.\dot{\mathbf{r}} \qquad (8)$$

Multiple forces

With multiple forces on particles, the net work done is the sum of each force-displacement contribution

$$W = \sum_i \mathbf{F}_i.\boldsymbol{\delta}\mathbf{r}_i \qquad \text{or} \qquad W = \int \mathbf{F}.d\mathbf{r}$$

Kinetic energy

From the momentum equation (Newton's 2nd law), we can derive the work done to put a body in motion:

$$\begin{aligned} W &= \int \mathbf{F}.d\mathbf{r} = \int m\frac{d\dot{\mathbf{r}}}{dt}.d\mathbf{r} \\ &= \int m\frac{d\mathbf{r}}{dt}.d\dot{\mathbf{r}} = \int m\dot{\mathbf{r}}.d\dot{\mathbf{r}} \\ &= \frac{1}{2}m\,\dot{r}^2 \end{aligned} \tag{9}$$

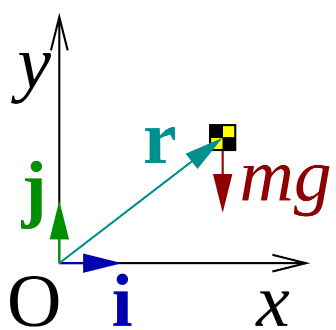
Work that leads to a change in **kinetic energy** is

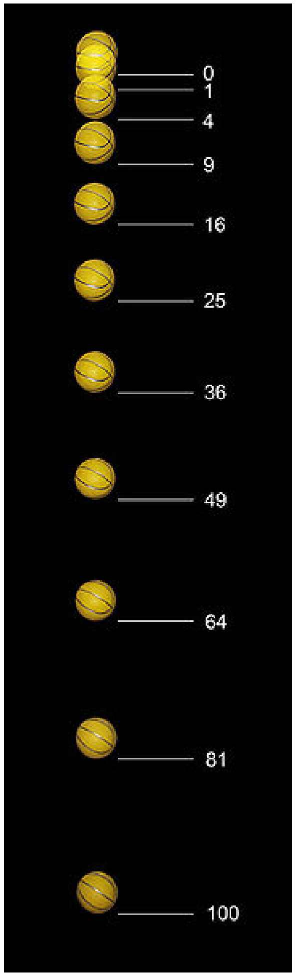
$$W = \int_{t_0}^{t_1} \mathbf{F}.d\mathbf{r} = \left[\frac{1}{2}m\dot{r}^2\right]_{t_0}^{t_1} = \frac{1}{2}m\,\dot{r}_1{}^2 - \frac{1}{2}m\,\dot{r}_0{}^2$$

Gravitational energy

Work done by gravity on an object moving in a constant gravitational field is

$$\begin{aligned} W &= \int \mathbf{F}.d\mathbf{r} \\ &= \int -mg\,\mathbf{j}.d\mathbf{r} \\ &= -mg \int \mathbf{j}.\,(dx\mathbf{i} + dy\mathbf{j}) \\ &= -mg \int dy \\ &= -mg\,\delta y \end{aligned} \qquad (10)$$





Summary of Dynamics (1)

- Introduction to dynamics
  - Newton's laws
- Linear momentum
- Work and power
  - sum of force-displacement contributions
  - power as rate of work
- Energy
  - kinetic energy
  - gravitational energy

Preparation for Dynamics (2)

- **Conservation of energy**

- In mechanics, what is a conservative force?

- Conservative forces

  – find two examples of a conservative force

- Non-conservative forces

  – find two examples of a non-conservative force