EE1.el3 (EEE1023): Electronics III

Mechanics lecture 4

Kinematics, motion of a particle

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www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3

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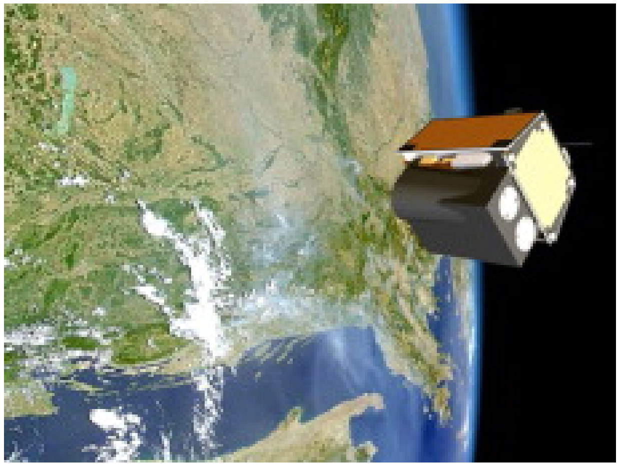
Kinematics (1)

- Motion of a particle
- Coordinate systems
- Displacement
- Velocity
- Acceleration



Motion of a particle

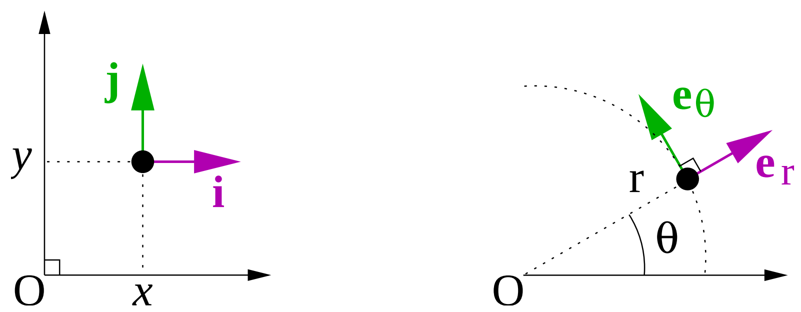
- How can you describe the position of a point in space?
  - in cartesian coordinates
  - in polar coordinates
- What are the relationships between position, velocity and acceleration?
  - write mathematical expressions of the relationships
  - in both directions

- Sketch an example of position, velocity and acceleration over time

Coordinate systems

- Two dimensional (2D) coordinate systems:

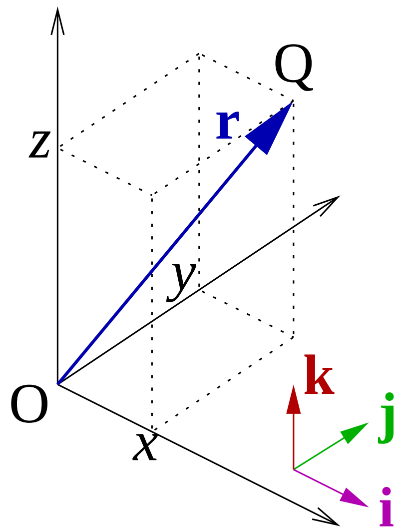
Cartesian Polar



- Three dimensional (3D) coordinate systems:
  - Cartesian $(x,\ y,\ z)$
  - Cylindrical $(r,\ \theta,\ h)$
  - Spherical $(r,\ \theta,\ \phi)$

Properties of displacement

- Components



$$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (1)$$

- Distance is the magnitude of displacement

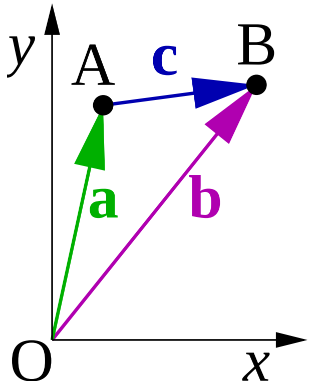
$$r = |\mathbf{r}| = \sqrt{x^2 + y^2 + z^2} \quad (2)$$

- Direction of displacement

$$\mathbf{e}_r = \frac{\mathbf{r}}{|\mathbf{r}|} = \frac{x}{r}\mathbf{i} + \frac{y}{r}\mathbf{j} + \frac{z}{r}\mathbf{k} \quad (3)$$

Change in displacement

- Displacement from A to B:



$$\mathbf{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

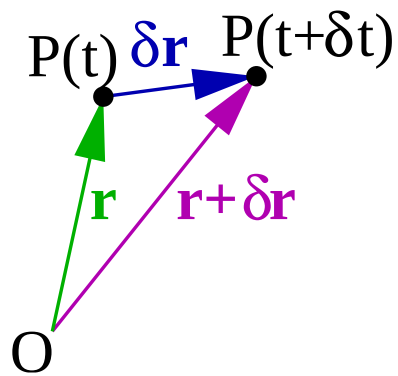
$$\mathbf{c} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

In three dimensions, the vector equation becomes

$$\begin{aligned} \mathbf{c} &= \mathbf{b} - \mathbf{a} \\ &= (b_x\mathbf{i} + b_y\mathbf{j} + b_z\mathbf{k}) - (a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}) \\ &= (b_x - a_x)\,\mathbf{i} + (b_y - a_y)\,\mathbf{j} + (b_z - a_z)\,\mathbf{k} \qquad (4) \end{aligned}$$

Velocity

- Velocity is change in displacement over time, $\mathbf{v} = \boldsymbol{\delta}\mathbf{r}/\delta t$



$$\mathbf{r}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \mathbf{r}(t+\delta t) = \begin{pmatrix} x+\delta x \\ y+\delta y \\ z+\delta z \end{pmatrix}$$

$$\boldsymbol{\delta}\mathbf{r} = \mathbf{r}(t+\delta t) - \mathbf{r}(t) = \begin{pmatrix} \delta x \\ \delta y \\ \delta z \end{pmatrix}$$

- Using Newton's notation, the instantaneous velocity is the rate of change of displacement:

$$\dot{\mathbf{r}}(t) = \frac{d\mathbf{r}}{dt} = \lim_{\delta t \to 0} \frac{\boldsymbol{\delta}\mathbf{r}}{\delta t} \qquad (5)$$

Properties of velocity

- **velocity components** in fixed coordinate frame

$$\dot{\mathbf{r}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \dot{x}\,\mathbf{i} + \dot{y}\,\mathbf{j} + \dot{z}\,\mathbf{k} \tag{6}$$

- **speed** is the magnitude of velocity

$$\dot{r} = |\dot{\mathbf{r}}| = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \tag{7}$$

- **direction** of velocity

$$\mathbf{e}_{\dot{r}} = \frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} = \frac{\dot{x}}{\dot{r}}\mathbf{i} + \frac{\dot{y}}{\dot{r}}\mathbf{j} + \frac{\dot{z}}{\dot{r}}\mathbf{k} \tag{8}$$

Acceleration

- the rate of change of velocity gives the instantaneous acceleration:

$$\ddot{\mathbf{r}}(t) = \frac{d\,\dot{\mathbf{r}}}{dt} = \frac{d^2\,\mathbf{r}}{dt^2} \tag{9}$$

- in a fixed coordinate frame,

$$\ddot{\mathbf{r}}(t) = \begin{pmatrix} \frac{d^2\,x}{dt^2} \\ \frac{d^2\,y}{dt^2} \\ \frac{d^2\,z}{dt^2} \end{pmatrix} = \begin{pmatrix} \ddot{x}(t) \\ \ddot{y}(t) \\ \ddot{z}(t) \end{pmatrix} \tag{10}$$

Properties of acceleration

- **acceleration components**

$$\ddot{\mathbf{r}} = \ddot{x}\,\mathbf{i} + \ddot{y}\,\mathbf{j} + \ddot{z}\,\mathbf{k} \tag{11}$$

- **magnitude** of acceleration

$$\ddot{r} = |\ddot{\mathbf{r}}| = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} \tag{12}$$

- **direction** of acceleration

$$\mathbf{e}_{\ddot{r}} = \frac{\ddot{\mathbf{r}}}{|\ddot{\mathbf{r}}|} = \frac{\ddot{x}}{\ddot{r}}\,\mathbf{i} + \frac{\ddot{y}}{\ddot{r}}\,\mathbf{j} + \frac{\ddot{z}}{\ddot{r}}\,\mathbf{k} \tag{13}$$

Example: problem statement

- A cricket ball travels through the air with a displacement that varies over time t according to

$$\mathbf{r}(t) = x\,\mathbf{i} + y\,\mathbf{j} \tag{14}$$

where $x(t) = t$ and $y(t) = 4t - t^2$.

  - Find $\mathbf{r}(t)$ for integer values of $t = \{0, 1, 2, 3, 4, 5\}$.
  - Find velocity $\dot{\mathbf{r}}(t)$ at $t = \{0, 1, 2\}$.
  - Find the acceleration $\ddot{\mathbf{r}}(t)$ at $t = 1$ and its direction.

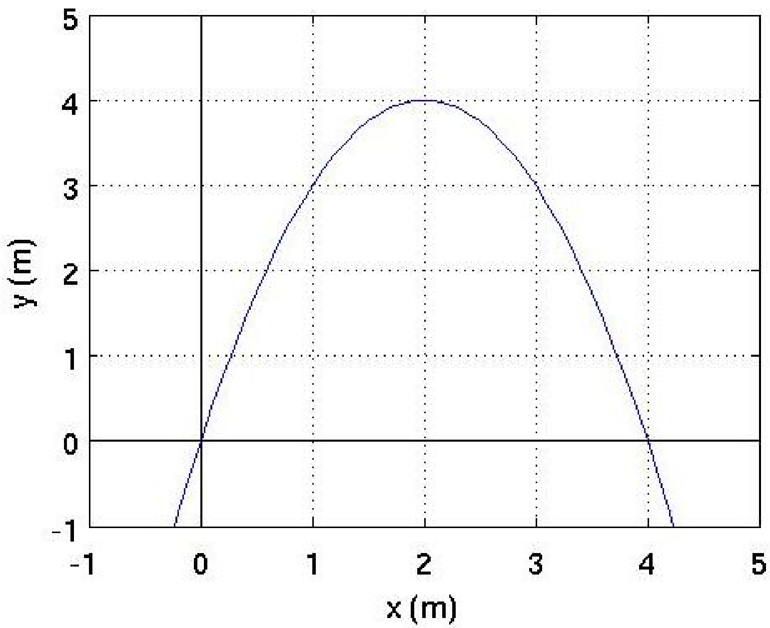
Example: displacement

- First, we can write down the expression for displacement

$$\mathbf{r}(t) = t\,\mathbf{i} + \left(4t - t^2\right)\mathbf{j} \tag{15}$$

and calculate x and y coordinates for values of t:

$x = t$	$4t$	$-t^2$	y
0	0	-0	0
1	4	-1	3
2	8	-4	4
3	12	-9	3
4	16	-16	0
5	20	-25	-5



Example: velocity

- To solve for velocity, we can differentiate the expression for the displacement:

$$\begin{aligned}\dot{\mathbf{r}}(t) &= \frac{d}{dt}\mathbf{r}(t) = \frac{d}{dt}\Big(t\mathbf{i} + \Big(4t - t^2\Big)\,\mathbf{j}\Big) \\ &= \mathbf{i} + (4 - 2t)\,\mathbf{j} \qquad (16)\end{aligned}$$

  or, in other words, we have $\dot{x} = 1$ and $\dot{y} = 4 - 2t$.

- Hence, we get the values for velocity at $t = \{0, 1, 2\}$:

$$\dot{\mathbf{r}}(0) = \mathbf{i} + 4\mathbf{j}$$

$$\dot{\mathbf{r}}(1) = \mathbf{i} + 2\mathbf{j}$$

$$\dot{\mathbf{r}}(2) = \mathbf{i}$$

Example: acceleration

- Similarly for acceleration, we can differentiate the expression for the velocity,

$$\begin{aligned} \ddot{\mathbf{r}}(t) &= \frac{d}{dt}\dot{\mathbf{r}}(t) = \frac{d}{dt}(\mathbf{i} + (4 - 2t)\,\mathbf{j}) \\ &= -2\mathbf{j} \end{aligned} \tag{17}$$

  or, in other words, we have $\ddot{x} = 0$ and $\ddot{y} = -2$.

- So, we get the value for acceleration at $t = 1$:

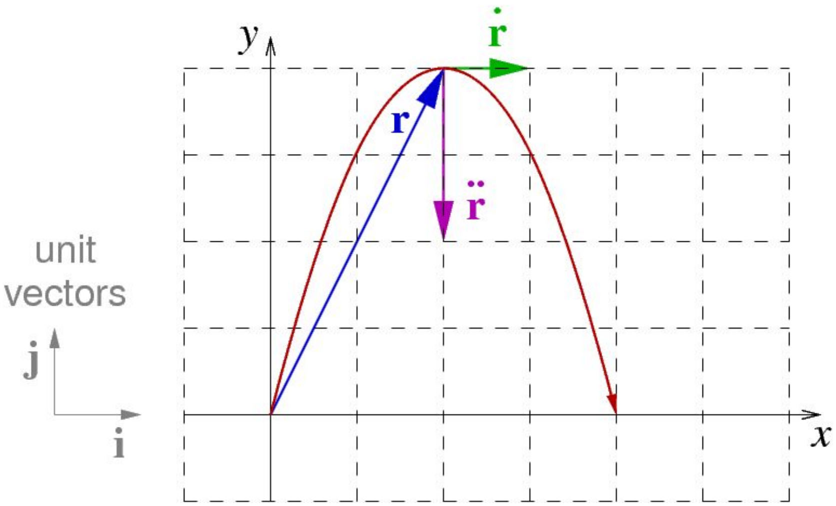
$$\ddot{\mathbf{r}}(1) = -2\mathbf{j}$$

- The direction of the acceleration is downward:

$$\mathbf{e}_{\ddot{r}} = \frac{\ddot{\mathbf{r}}}{|\ddot{\mathbf{r}}|} = \frac{-2\mathbf{j}}{2} = -\mathbf{j}. \tag{18}$$

Summary of Kinematics (1)

- **Motion of a particle**
- Coordinate systems
  - Cartesian and polar
- Displacement
  - Distance and direction of position
- Velocity
  - Rate of change of displacement
- Acceleration
  - Rate of change of velocity



Preparation for Dynamics (1)

- Work through the exercises given on the web site:

  www.ee.surrey.ac.uk/Teaching/Courses/ee1.el3 > Exercises

- **Forces in motion**

- Newton's laws of motion

  – what are they?

- Illustrate the laws

  – draw three sketches to illustrate each law