The Microstrip Ring Resonator for Characterising Microwave Materials

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Summary

The edge coupled microstrip ring resonator is commonly used to determine microwave substrate properties, in particular the dielectric constant and loss tangent. It is also well known that under certain circumstances, a microstrip ring circuit can act as a narrowband antenna. Normally radiation losses are neglected when using ring resonators to determine substrate parameters, but it can be shown that for thick substrates with low permittivity and loss, radiation is the most significant loss mechanism. This work has studied the microstrip ring resonator in detail and has developed accurate methods of linking measured data with the material properties through the use of an equivalent circuit. Experiments have been carried out on materials with known properties to test the validity of the theory and good agreement has been obtained. The detailed study of the radiation losses of ring resonators is new, and both theory and experimental results have been the subject of a conference presentation. In addition, a more theoretical analysis has been submitted to the IEEE Transactions on Microwave Theory and Techniques.

Following on from the analysis of the ring resonator, a careful study of the properties of a new polymer circuit material is proposed using these techniques. It is expected that this material has useful microwave properties in the millimetre wave band. Analyses of materials at these high frequencies are scarce, and new techniques are proposed to measure the material properties at frequencies greater than those where microstrip can be used.

A new technique for calculating the attenuation produced by surface roughness features of metallic conductors used in planar circuits is also proposed, as when the dimensions of these features approach the skin depth, attenuation increases significantly. This additional loss is currently poorly modelled, but an enhanced theory would offer significant practical utility.

In addition to reviewing existing work in the field of microstrip ring resonators, a plan for future research is given which is intended to define the remaining two years of the PhD program.
1 Introduction

The production of high frequency and high performance radio frequency circuits is critically dependent on good performance from the circuit board material on which the circuit is fabricated. Microwave circuits have traditionally been fabricated on ceramic substrates such as Alumina which offers both low dielectric loss and high dielectric constant allowing compact circuits to be built. Unfortunately, Alumina is a fairly dense material, and is brittle and difficult to machine. In high purity form, it is also expensive. Multilayered circuits are also time consuming to fabricate on Alumina, as they require repeated firing. Several new materials technologies have been introduced to try to overcome some of these difficulties and have resulted in the development of new types of substrates. Some examples include low temperature cofired ceramic (LTCC) which is a multilayered technology and allows multilayered circuits to be printed on different layers of substrate which are then stacked, laminated and fired together. Another class of material is based on plastic polymer based substrates. These offer a very low cost of production through traditional circuit manufacturing processes, and can also be laminated to make multilayered circuits. A recent example of this material is liquid crystal polymer (LCP) which is a thermosetting plastic that can be injection moulded into any shape.

The introduction of these new materials with desirable mechanical properties requires that their electrical performance be characterised before they can be used in microwave circuits. The most important properties are the dielectric constant of the material and the loss tangent (which is related to the conductivity). Both of these parameters are frequency dependent. Traditional methods of measuring the dielectric constant are only useful at low frequencies (1MHz) and are based on constructing capacitors using the material as a dielectric. The capacitance is directly proportional to the dielectric constant. At higher frequencies, waveguides can be used by filling the guide with the material to be measured and then determining the propagation velocity, which is inversely proportional to the square root of the dielectric constant. This technique requires accurately machined, samples of material, which might not be compatible with a process for manufacturing large thin sheets used for circuit board manufacture. Another microwave technique for measuring material properties is via the use of a split post resonator. This comprises a resonant cavity of which the microwave transfer function is measured. A small sample of the dielectric material is introduced to the cavity, and the perturbation of the resonance can be related to the material properties. The cavity can only be used at a small number of frequencies, but provides very accurate and repeatable spot frequency measurements of a material. At high frequencies, the cavity is very small and requires precision machining of the sample.
Because the materials under consideration are designed for producing printed circuits, and the properties of the material influence the printed circuit behaviour, it seems reasonable to assume that it is possible to learn about the substrate by measuring some type of test circuit. The test circuit would comprise some pattern for which it is possible to accurately calculate the performance based upon estimates for the substrate parameters. This circuit could then be measured, and by an analytical process, the measured data used to fit the substrate parameters to the theoretical model. Perhaps the most straightforward test pattern consists of a transmission line constructed using microstrip, coplanar waveguide or similar technique. The propagation factor of this transmission line can be measured with a vector network analyser and this data used to estimate the substrate loss and dielectric constant. The challenge in designing a test pattern lies in being able to accurately account for all of the non ideal behaviour in the circuit. Mismatch errors, radiation, finite thickness conductors and dispersive propagation all increase the complexity of the analytical model. This may make the test circuit unduly sensitive to a parameter which might be difficult to accurately control, for example the substrate thickness or sharpness of metal edges. Finally, it would be desirable that the test pattern exhibit broad band performance so that the substrate could be characterised over a wide range of frequencies. For example, some authors have used a microstrip antenna circuit to measure the dielectric constant, but this only yields the dielectric constant at one frequency, however measuring the propagation velocity of a transmission line can be carried out at an arbitrary frequency resolution over more than a decade frequency range.
2 Purpose of project

This project aims to develop some printed circuit test structures for the purpose of characterising the electrical performance of a dielectric substrate. The structures will be designed in such a fashion as to be able to gather data at a wide range of frequencies from a single circuit. Instrumentation is available within the department for network analysis between 45MHz and 220GHz. The use of printed circuits for characterising materials is not new, however this project will develop the circuits with the specific intention of improving the accuracy of the measurements. Because of the relative scarcity of instrumentation capable of measuring electrical parameters at frequencies above 110GHz, many existing literature studies are limited to this frequency range. Because some materials still exhibit acceptable performance at these high frequencies, it is considered valuable to be able to develop test circuits suitable for measuring circuits at even higher frequencies, where applications still exist for millimetre wave circuits. This project will consider circuits at frequencies above 110GHz where it is considered that macroscopic features in the circuits, such as the conductor thickness will become significant.

The output of this project will be the designs of several planar test structures for the purposes of measuring the broadband variation of dielectric constant and dielectric loss tangent at microwave and millimetre wave frequencies. The structures will be analysed theoretically so as to provide a robust link between the measured data and the analytical performance.
3 Background Review

Initial studies showed that the microstrip ring resonator is a very popular technique for measuring the dielectric constant, and it is straightforward to find many examples of their use in text books, journal papers and conference proceedings. This study therefore initially concentrated on the microstrip ring resonator technique. The microstrip ring resonator has found three main uses.

1. As a resonator with several well defined resonant frequencies for the purposes of measuring microstrip properties
2. As a narrowband antenna, possibly operating at a higher mode where the bandwidth is wider and somewhat more useful
3. As an element in a filter, either as a simple resonator, or by modifying the symmetry of the ring to introduce extra closely spaced resonances

This work concentrates on the ring resonator technique as a method of measuring microstrip properties, which can be used to infer information about the dielectric. The radiation behaviour is also found to be important, as this has implications on the loss in a ring resonator.

To use the ring for microstrip measurements, an annular ring is constructed into which microwaves are injected. When the ring is an integer number of wavelengths long, a standing wave pattern is set up, and the ring displays resonant characteristics. The microstrip ring resonator was used extensively in the study of microstrip dispersion due to the ease of measuring the effective dielectric constant. The ring resonator was first presented by Troughton in 1969 [1] who described the new technique and plotted graphs of the effective dielectric constant of various microstrip lines on Alumina. He comments on the repeatability of the measurements, however due to the limited knowledge of microstrip dispersion in 1969, it was not possible to relate the effective dielectric constant to the material dielectric constant except at very low frequencies. In 1971, Wolff and Knoppik [2] revisited the ring resonator and considered the effects of the width of the microstrip on the ring. They devised an electromagnetic model of the resonator which was able to account for the width of the microstrip track on the resonant frequency. They concluded that narrow tracks suffered less from the effects of dispersion, but they were still unable to account for dispersion. Critically, they discovered that for a given track width, the resonant frequency depended on the radius of the ring above and beyond that expected from the total track length alone, and thus concluded that the curvature of the ring affected the resonant frequencies. Wu and Rosenbaum [3] published a very widely cited chart showing the existence of higher order modes that could exist across the width of the resonator. They used an empirical correction in the width of the ring which when substituted into the electromagnetic equations of Wolff and Knoppik [2] attempted to account for the microstrip fringing fields. In 1975, Kompa and Mehran [4] introduced the planar waveguide model for
microstrip. They proposed substituting a parallel plate waveguide for an open microstrip line. This waveguide has electric walls at the top and bottom, and magnetic sidewalls. This simplified model made calculation of microstrip behaviour easier. Their waveguide has the same height and characteristic impedance as the microstrip and is uniformly filled with a dielectric with a dielectric constant equal to that required to make the velocity of propagation the same on the two different guides. They noted that at higher frequencies the electric field in a microstrip was increasingly concentrated in the dielectric, therefore the width of the equivalent planar waveguide is frequency dependent and decreases, approaching the width of the microstrip at high frequencies. In 1976 Owens [5] incorporated this planar waveguide model of microstrip into the ring resonator and was able to essentially eliminate the curvature effect. After Owens, no further attempt to improve the accuracy of the resonant ring technique based on microstrip theory seems to have occurred. However, Owens provided a link between the ring resonator and microstrip theory, so advances in microstrip design could be used to enhance ring resonator measurements. Eventually, analytic expressions for calculating the effective dielectric constant due to dispersion effects were introduced, and accurate expressions for the effective dielectric constant based on the geometrical details of the microstrip line were created.

The most recent design equations for microstrip seem to be those of Hammerstad and Jensen [6] These are widely cited in current text books, and claim very good accuracy. They first present an equation for calculating the impedance of a microstrip line in a homogenous medium. This expression is stated to have an accuracy of better than 0.03% for all practical strip dimensions. A correction factor for the microstrip width is given to accounting for conductors of non-zero thickness. The accuracy and validity of these corrections is not stated. An expression is then given for calculating the (static) effective dielectric constant for a non homogenous (i.e. practical) microstrip based upon the dimensions and dielectric constant. The static impedance of the microstrip line can then be calculated by reducing the impedance of the homogenous line by the square root of the effective dielectric constant.

Many models for accounting for the frequency dependence of the effective dielectric constant exist. This frequency dependence causes the dispersive behaviour of the microstrip. The most recent and widely cited models for microstrip dispersion are by Kobayashi [7] and Kirschning and Jansen [8]. Both of these works present complicated curve fitted expressions, and Kobayashi states that the equations are not physically well understood, but that accurate predictions of the effective dielectric constant result. An important observation can be made regarding the effective dielectric constant. The dispersive equations allow the velocity of propagation and the guide wavelength on the line to be derived from
\[ V_p(f) = \frac{c}{\sqrt{\varepsilon_{\text{eff}}(f)}} \]
\[ \lambda_g(f) = \frac{c}{f \sqrt{\varepsilon_{\text{eff}}(f)}} \]

Because the propagation in microstrip is not pure TEM, and there are small transverse components of the surface current density, and hence longitudinal variations in the magnetic field, the exact characteristic impedance is not well defined. Kirschning [9] gave a very lengthy procedure for calculating \( Z_0(f) \) which involves 17 curve fitted coefficients, although a simpler expression is given by Hammerstad and Jensen [6].

\[
Z_0(f) = Z_0(0) \frac{\varepsilon_{\text{eff}}(0)}{\varepsilon_{\text{eff}}(f)} \frac{\varepsilon_{\text{eff}}(f) - 1}{\varepsilon_{\text{eff}}(0) - 1}
\]

Although the characteristic impedance of the microstrip affects the effective width of the ring resonator via the planar waveguide model, the ring resonant frequency is not very sensitive to the width, so the primary factors which affect the resonant frequencies are the effective dielectric constant and the physical length of the ring.

Development of more accurate expressions for microstrip properties appears to have stopped, as the closed form expressions claim to offer accuracies of better than 1% [6], [7], [8] which appear accurate enough for most practical situations. Computer based electromagnetic solutions would be required for more accurate predictions, and given the widespread use of such systems, it appears not to be thought necessary to create vastly complex closed form expressions for estimating microstrip parameters which would require a computer to evaluate anyway.

A parallel study into the use of annular ring structures for making microstrip antennas will also be presented, as it will be shown that this becomes useful for calculating losses in the microstrip ring resonator. Basic electromagnetic theory [10] predicts that accelerating a charge will result in radiation. Therefore it logically follows that any alternating current flowing around a curve will radiate. The microstrip disc has been used as a radiating antenna element for many years, and in 1980 Bahl, Stuchly and Stuchly [11] developed an annular ring antenna for medical applications. This antenna used the patient’s body as a dielectric cover for a microstrip antenna and radiated at 2.5GHz. The presence of tissue with different dielectric properties allowed various different modes in the antenna to be excited. Wood [12] also considered curved microstrip lines as antennas and devised analytical techniques for their analysis. He proposed using surface magnetic current sources flowing on either side of the microstrip at a width given by the planar waveguide equivalent width. Various authors [13], [14], [15] have analysed the annular ring, especially at higher (radial) modes, where they become quite effective antennas. Bhattacharyya and Garg [16] also analysed the antenna, but using an interesting equivalent circuit model employing lossy transmission lines.
Perhaps the most useful work is that of Bahl and Stuchly [17] which summarises work to date, and gives expressions for the far field electric field components. Most importantly, they demonstrate that theoretical antenna efficiencies in excess of 80% can be achieved by microstrip ring antennas.

Analysis of the ring resonator is complicated using the electromagnetic models, but may suffer from reduced accuracy using the transmission line model. Various authors have attempted to produce equivalent circuits for the ring when operating near to resonance. The equivalent circuit can also be used to examine the behaviour of the coupling gap, which even Troughton [1] was aware influenced the ring behaviour. Yu and Chang [18] modelled the ring as a transmission line and modelled the ring and coupling gap as a capacitor network, with the ring capacitance being frequency dependent. This work built upon several studies of the equivalent circuit of the gap between two microstrip lines [19]. Hsieh and Chang [20] improved this circuit to consider the ring alone, and modelled it as a simple LCR circuit, although their circuit was only valid for a particular resonance, and they only presented data for the first resonant frequency. Again, the use of the microstrip research allows expressions for the loss due to both dielectric and conductor loss to be considered in the equivalent circuit. James and Hall [21] and Garg et al. [22] also present an equivalent circuit for the ring antenna with electromagnetic analysis.

Edwards and Steer [23] describe the ring resonator in their popular textbook and cite the key advantage of the ring resonator as being almost free of radiation loss. In addition, authors using the ring resonator to calculate the dielectric loss frequently neglect radiation. However, as the references show, the microstrip annular ring circuit can form an antenna, therefore a ring resonator should exhibit radiation loss (at least under certain circumstances), which would seriously affect loss measurements. This apparent disparity in the literature has been addressed by the author, and is the subject of a conference presentation [24].

A recent example of authors using the microstrip ring resonator is [25]. The authors use a microstrip transmission line analysis to determine the resonance frequency of a microstrip line fabricated using a thick film microstrip process, however only quasi-static microstrip design equations are used, and no attempt is made to include the effects of dispersion. Their resonator was built on an alumina substrate and used dielectric paste to make a thick film microstrip. Results are presented between 15GHz and 110GHz. In order to determine the thick film dielectric paste loss tangent, they considered losses due to the conductor surface roughness and dielectric heating as well as radiation and measured the Q factor of each resonance. An electromagnetic simulator was used to work out the loss excluding dielectric loss, then the supposed dielectric loss was deduced by subtracting the calculated loss from the measured loss. A similar approach was taken by [26] on a liquid crystal polymer substrate. They noted that the effects of dispersion could be reduced by using higher impedance (narrower) lines, but still did not include any corrections for its effect. They
conceded that using the ring resonator was difficult for measuring the loss tangent of the material due to the lack of accurate expressions for calculating conductor losses at millimetre wave frequencies. Radiation was only considered superficially, and only the coupling gaps were assumed to radiate. Heinola et al. [27] have also considered the use of a ring resonator to accurately characterise a FR4 substrate at frequencies below 10GHz. They take great care to isolate mechanical and environmental influences in the substrate, and control the temperature and humidity. However, they state without justification that radiation losses are minor, and only consider dielectric and conductor losses and their effect on the measured Q factor.

The ring resonator has also been used to characterise integrated circuit substrates. Chen et al. [28] describes resonators built using a thin film process on a silicon VLSI IC. The work uses a tightly coupled microstrip ring, and presents an equivalent circuit. The paper shows a low Q resonance at 28GHz indicating that the silicon substrate is conductive, and results in poor RF performance. The paper also describes a CPW resonator, which achieves similar results. Finlay, Jansen, Jenkins and Eddison [29] used a capacitively coupled ring resonator on GaAs in order to measure the effective dielectric constant and characterise line loss for different methods of printing conductors and passivation layers on a GaAs MMIC process. They estimated that the uncertainties in the measurement of the effective dielectric constant were better than 1% over a frequency range of 2GHz-24GHz.

Many examples of microstrip rings being used as resonators in filters exist. Their compact size and lack of end effects make them superior to straight resonators, and they can offer low radiation and high Q performance. Chang [30] and Navarro and Chang [31] have experimented with adding varactor diodes at strategic points around the ring to tune their resonant frequency. Such devices offer interesting potential as tank circuits in VCOs and tuneable filters. It has also been noted that two degenerate modes exist in the ring resonator at each resonance [2]. By introducing asymmetry into the ring, it is possible to split these modes so that they occur at different frequencies. This effect has been exploited to make band pass filters whereby a small perturbation in the form of a short stub is placed at an angle 135° around the ring, and the two feeds placed 90° apart. By controlling the size of the perturbation, the degree of separation of the resonances can be controlled, and hence the filter bandwidth. This so called “dual mode resonance” effect is exploited by Huang and Cheng who demonstrated a square ring resonator based bandpass filter at 1.4GHz [33].
4 Work to date

4.1 Introduction

Three separate areas of work have been completed to date using ring resonators. An investigation into the (unknown) dielectric constant of a new polymer PCB substrate has been carried out over the frequency range 10GHz-110GHz. These measurements have been compared with a broadband measurement of the velocity of propagation of a CPW line on the same substrate. This work has been submitted to the IEEE Microwave and Wireless Components Letters [36]. A second area of study concentrated on the radiation losses of ring resonators, and combined theory from ring antennas and ring resonators. This study calculated and measured the radiation efficiency of resonators fabricated on a thick substrate with low permittivity and found that radiation loss was significant. Results from this work have been accepted for oral presentation at the 39th International Microelectronics Symposium [24]. A final area of study was that of the broadband accuracy of the ring resonator for the purposes of determining the dielectric constant. Resonators were constructed on alumina and the resonance frequencies of more than 10 modes carefully measured. By the use of an equivalent circuit model which is able to de-embed the ring resonator from the feed network, and taking into account non ideal behaviour such as dispersion and radiation, agreement between the measurements and theory of better than 0.5% was obtained. This work has been submitted to Microwave Theory and Techniques Transactions [37] for publication.

4.2 Polymer measurements

For this study, it was desired to determine the dielectric constant of the polymer substrate over a wide range of frequencies. A sample of polymer substrate was used with 17um thick rolled copper conductor and a 127um thick dielectric layer. This layer is a glass fibre weave, bonded with polymer resin. A photomask was produced with several a rings with mean radius 3000um and width 200um. The ring was coupled to microstrip feeds by gaps of various widths as it is known that tight coupling of the microstrip feed to the ring affects the resonant frequencies. The feedlines were 200um wide and the overall circuit comprising five rings and two straight microstrip lines is 50mm x 50mm.
Figure 1 - Photo mask for polymer ring resonators

Figure 1 Also shows an enlarged view of the CPW-microstrip transition. The diagram shows the top metal layer. The bottom surface is solid conductor (groundplane) and hence the circuit supports microstrip modes. As the circuit is intended to be used at millimetre wave frequencies, connections to the instrumentation are achieved with 100um pitch GSG coplanar probes. The ends of the microstrip lines were terminated in grounding pads for a coplanar waveguide probe station. Vias are made between the top and bottom layers using conducting silver paint.

Because of the narrow gaps required for the test circuit, it was necessary to thin the copper layers. This was achieved by etching the untreated copper / polymer laminate in ferric chloride for five minutes. The laminate was then coated in spray on UV photoresist, and the sensitised board exposed in the normal method. The circuit was then etched using ferric chloride and the photoresist removed with acetone.

Measurements with a Dektak II profilometer showed that the copper was thinned to a thickness of 8um, but due to undercutting, all copper features are receded by 10um. The ring and feedline are therefore 180um wide. Because of the thin substrate, coupling between the microstrip and ring was extremely weak for gaps wider than the thickness of the substrate, and no transmission measurements could be made. The only rings which yielded useful measurements were those with a 70um or 120um gap (these widths include the effects of undercutting).
Figure 2 shows a plot of the insertion loss of the rings. An HP8510XF network analyser system was used with 100um pitch coplanar waveguide probes. Nine resonant modes are visible, spaced by approximately 11GHz. In addition to the expected resonant modes, there are some parasitic peaks on the graph, particularly around 50GHz and 92GHz. These are though to be due to the feed line and CPW-microstrip transition resonating. Insertion loss is high above 100GHz, and the resonances were not all visible on all of the circuits. A plot of the transmission characteristics of the straight through feed connection can also be seen. The ripples in the response are due to the line resonating as the impedance is around 75\(\Omega\) and is mismatched to the network analyser. No special attempt was made to optimise the performance of the CPW-microstrip transition as it does not affect the resonant frequencies.

By narrowing the frequency span, the peak in the \(S_{21}\) was recorded around each resonance. It is important to note that the some of the modes split by a few 10s of MHz and the exact peak was not easily identifiable. This limits the ultimate accuracy of the measurement. It is thought that this is due to tiny asymmetries in the ring which allow waves propagating around the ring in opposite directions to have slightly different resonant frequencies. The asymmetries are of the form of mechanical defects in the substrate and scratches on the surface. As this mechanism affects the accuracy of which the resonant frequency can be determined, and the effects of unavoidable defects on the samples increase with frequency, it is proposed that these effects are studied more carefully.
The rings were analysed using the transmission line model of the ring. This states that the $n^{th}$ resonance occurs at

$$f_n = \frac{nc}{2\pi r \sqrt{\varepsilon_{\text{eff}}(f)}} \quad (4.1)$$

Where $r$ is the mean radius, $c$ is the speed of light in a vacuum and $\varepsilon_{\text{eff}}(f)$ is the frequency dependent effective dielectric constant.

Measurements were taken on three separate rings and the effective dielectric constant is plotted in Figure 3.

![Figure 3 – Variation of effective dielectric constant with frequency for polymer rings](image)

The dispersive nature of the microstrip ring is visible as the effective dielectric constant is seen to increase slightly with frequency. With the exception of one point, the very close agreement in resonant frequencies between the rings with wide and narrow gaps shows that the gap is wide enough not to be significantly affecting the resonance by loading the rings. The exact error bounds on the measurements are not known accurately, however most of the points from the three independent experiments lie close together which increases confidence in the accuracy.

By using the standard microstrip design equations of Hammerstad and Jensen [6] it is possible to calculate the static ($f = 0$) effective dielectric constant for the microstrip ring. This calculation can also incorporate the thickness of the conductor for improved accuracy. Using the methods described by Kirschning and Jansen [8] or Kobayashi [7] it is then possible to include the effects of dispersion and calculate an improved frequency dependent effective dielectric constant based upon the microstrip dimensions and static effective dielectric constant. Using straightforward iterative techniques, given the microstrip dimensions, effective dielectric constant and frequency,
the material dielectric constant at each frequency can be inferred. This is plotted for the polymer measurements in Figure 4.

![Graph showing variation of actual dielectric constant with frequency for polymer rings](image)

**Figure 4 – Variation of actual dielectric constant with frequency for polymer rings**

The ring coupled with the wide gaps (120um) had a greater insertion loss than the rings coupled with the 70um gaps. This loss was too high to allow the resonances above 90GHz to be clearly seen and measured.

In the letter [36] the ring resonator measurements of the dielectric constant were compared with those made by an independent method using coplanar waveguide meander line techniques and the agreement is found to be better than 1.5%. The results show that the dielectric constant of the polymer substrate decreases slightly with frequency in the frequency range considered.

### 4.3 Electromagnetic analysis

Experimental work relating to this study has been presented at a conference [24], but more theoretical investigations are the subject of a paper submitted to MTT [37].

The microstrip ring is initially modelled using a cavity model. In this model, a microwave cavity is assumed to be formed between the printed ring conductor and the groundplane. The conductor is assumed to be perfect and fringing fields are neglected so that the electric field is confined to the dielectric, between the conductor layers. The geometry is set up in the cylindrical coordinate system shown in Figure 5.
Assuming $h \ll \lambda_g$, the electric field in the z direction is almost constant. Because the conductor is very thin compared to the height of the substrate, there is very little current flowing in the z direction so there can be no $H_z$ magnetic component. Additionally, continuity requires that all current density must be continuous, therefore there can be no current at the edge of the strip normal to the strip edge. This in turn implies that at the strip edge, there must be no tangential component of the magnetic field. Thus the magnetic field must exit the edge of the patch normal to the edge, hence an imaginary magnetic conducting walls can be placed vertically at the edge of the conducting sheets. Similarly, as the conductor is assumed to be perfect, no electric field exists across either conductor. These assumptions allow the boundary conditions imposed by the structure to be simplified and Maxwell’s equations can be solved in the region.

Since there is no variation of the electric field in the z direction, the field distribution is TM in the z direction. Various modes can exist, of the form $TM_{nm}$ where $n$ corresponds to variation in the $\phi$ direction (around the ring) and $m$ corresponds to variation across the width of the ring, $\rho$. By solving the wave equation in cylindrical coordinates as shown, subject to the boundary conditions solutions exist of the form: [17]
\[ E_z = E_0 \left[ J_n'(k\rho)Y_n'(ka) - J_n'(ka)Y_n'(k\rho) \right] \cos n\phi \]

\[ H_\rho = \frac{j\omega \varepsilon_r \varepsilon_0}{k^2} \frac{\partial E_z}{\partial \rho} \]

\[ H_\phi = \frac{j\omega \varepsilon_r \varepsilon_0}{k^2} \frac{\partial E_z}{\partial \phi} \]

\[ k = \frac{2\pi \sqrt{\varepsilon_r}}{\lambda_0} \]

(4.2)

Where \( \lambda_0 \) is the free space wavelength, \( J_n \) is the first kind of Bessel function of order \( n \), \( Y_n \) is the second kind of Bessel function of order \( n \) and prime(') represents differentiation. \( k \) is the circular wavenumber, or equivalently, the imaginary part of the phase propagation factor.

A well known eigen equation results from applying the boundary conditions and its solution represents resonance.

\[ J_n'(kb)Y_n'(ka) - J_n'(ka)Y_n'(kb) = 0 \]

(4.3)

Because the microstrip width is much narrower than the length, circumferential modes occur at much lower frequencies than radial modes.

The predictions of the resonant frequency by these equations is too low by a factor of several percent as they do not account for the fringing fields. By using the planar waveguide model [4], a correction factor is made to the width of the microstrip. It is found, particularly for wide rings, that the use of the physical ring width produces inaccurate predictions of the resonant frequency.

Several different correction terms exist in the literature for correcting the width of ring to account for curvature using the planar waveguide model.

All of the corrections are based around modifying the inner and outer radii as in (4.4).

\[ a_c = a - \left[ w_{\text{eff}}(f) - w \right] / 2 \]

\[ b_c = b + \left[ w_{\text{eff}}(f) - w \right] / 2 \]

(4.4)

At least three different expressions exist for \( w_{\text{eff}} \) although none of them appear to have been derived from first principles, but instead have been proposed to fit measurements.

Kompa and Mehran [4], in presenting the Planar waveguide model for microstrip in 1975 suggest

\[ w_{\text{eff}}(f) = w + \frac{w_{\text{eff}}(0) - w}{1 + f / f_s} \]

(4.5)

\[ f_s = \frac{c}{2w\sqrt{\varepsilon_r}} \]

Owens proposed a correction for this expression to improve agreement with experiments on ring resonators. His modification is stated to “provide satisfactory curvature correction” [5]. This expression is also quoted by James and Hall [21]
In 1991 Bahl and Stuchly [17] stated simply for microstrip

\[ w_{\text{eff}} = \frac{h\eta_0}{Z_0\sqrt{\varepsilon_{\text{eff}}}} \]  

(4.7)

Although in [22], this was extended to

\[ w_{\text{eff}}(f) = \frac{h\eta_0}{Z_0(f)\sqrt{\varepsilon_{\text{eff}}(f)}} \]  

(4.8)

Thus allowing various accurate frequency dispersive expressions for \( Z_0(f) \) and \( \varepsilon_{\text{eff}}(f) \) to be used. This work has assumed that this final expression is the most accurate, despite not being specifically related to curved microstrip. It is clear that all of these different expressions developed over the years show that the effect of curvature on the ring resonator is not robustly understood. When the ring is resonating at higher modes, the effects of dispersion additionally come into play, and it is likely that several of the models are confused by trying to account for both curvature effects and dispersion effects at the same time, with the same expressions.

It will be demonstrated that the resonant frequency is insensitive to the width of the ring if it is very narrow. This is a particularly useful result, as it allows the use of the simpler transmission line model. Wu and Rosenbaum [3] provided a simplified expression for the solution of the electromagnetic eigenequation. In the limit that the inner ring radius increases to that of the outer radius, the resonance equation reduces to

\[ \left( k^2 - n^2 \right) J_n(kh)Y_n(kh) - Y_n(kh)J_n(kh) = 0 \]  

(4.9)

Since the second term is non zero

\[ \left( k^2 - n^2 \right) = 0 \]  

(4.10)

If the wavenumber, \( k \), is replaced with \( 2\pi/\lambda_g \) then

\[ n\lambda_g = 2\pi b \]  

(4.11)

The above expression [3] is true only in the limit \( a \) as tends \( b \). If the ring is narrow (rather than infinitely thin) then a better approximation is obtained by using the ring mean radius instead of the outer radius

\[ n\lambda_g = \pi(a + b) \]  

(4.12)

Which can be rearranged to give the resonant frequency
\[ f_n = \frac{nc}{\pi(a+b)\sqrt{\varepsilon_{\text{eff}}}(f'_n)} \] (4.13)

By comparing the evaluation of this last equation with an exact solution of the eigenequation (4.3) it is possible to see the effect of curvature on the resonance frequencies. This last equation is independent of curvature, so an infinitesimally narrow ring would display linearly spaced wavenumbers at resonance. The solution of these equations is the wavenumber which is related to the resonant frequency by the phase velocity on the material. Due to dispersion, the phase velocity is frequency dependent, although expressions for the effective dielectric constant exist in the literature. Ignoring the effects of dispersion, Figure 6 shows a plot of the solution of the eigenequation compared with the approximation. The ring width is normalised to the radius.

![Figure 6 – Effect of ring aspect ratio on resonant frequency](image)

A simple rule of thumb would seem therefore to be that the effects of curvature can be neglected for rings narrower than perhaps 0.15 times the mean radius. Stated differently, this means that the simple linear approximation to the resonant wavenumber, neglecting the width of the ring, is sufficiently accurate provided that the ring is narrower than 0.15 times the mean radius. It is important to note that the resonant frequency eigenequation is solved using the effective width of the ring, which is proportional to the substrate height, and may be considerably wider than the physical width. This further emphasises the need for narrow rings.

Therefore if the ring resonator is used purely for calculating the effective dielectric constant of a microstrip line on a material, a narrow ring width allows the simple linear estimation of resonant mode number to be used. For higher modes, the effects of dispersion must be taken into account, for example by using the equations of Kobayashi [7] or Kirschning and Jansen [8]. An additional advantage of using a narrow line is that many \( \text{TM}_{n1} \) modes can be observed as radial modes (\( \text{TM}_{n2} \text{ TM}_{n3} \text{ etc.} \)) occur at much higher frequencies. It is likely that the limiting factor for
using the ring resonator at very high frequencies is either the loss of the conductor or substrate, or
the substrate becoming electrically thick rather than undesirable radial modes. Additionally, the
radiation efficiency of the ring resonator is lower for narrow widths which will result in a higher Q
factor.

4.4 Equivalent circuit

Hsieh and Chang [20] analysed the input impedance of the ring resonator by assuming it to be
formed of two transmission lines connected in parallel. Figure 7 is based on a diagram from their
paper and shows the two lines.

![Figure 7 – Transmission line equivalent circuit](image)

By making simple extension of the theory of Hsieh and Chang, it is possible to apply their model at
higher modes. In this paper, the ring resonates when the ring length is one wavelength.

\[
l = \lambda_g
\]

(4.14)

Where \( l \) is the (electrical) length of the ring and \( \lambda_g \) is the guided wavelength. As the transmission
line (and ring) exhibits periodic behaviour, it is reasonable to assume that further resonances occur
when

\[
l = n\lambda_g
\]

(4.15)

The \( Y \) parameters of a transmission line of length \( l \) are

\[
y_{11} = y_{22} = Y_0 \coth \gamma l
\]

\[
y_{12} = y_{21} = -Y_0 \csch \gamma l
\]

(4.16)

If the ring is then constructed out of two parallel transmission line sections, with length \( l_1 \) and \( l_2 \),
and admittance \( Y_0 \) the overall \( Y \) parameters can be added to give

\[
Y = Y_0 \begin{bmatrix}
\coth(\gamma_{l_1}) + \coth(\gamma_{l_2}) & -\csch(\gamma_{l_1}) - \csch(\gamma_{l_2}) \\
-\csch(\gamma_{l_1}) - \csch(\gamma_{l_2}) & \coth(\gamma_{l_1}) + \coth(\gamma_{l_2})
\end{bmatrix}
\]

(4.17)

The input impedance of the ring is defined as the ratio of the input voltage to the input current with
no current flowing in port 2 (i.e. open circuit). That is
Since the second port is open circuit, it is possible to define $l_1 = l_2 = l/2$ and through some trigonometric manipulation the ring input impedance is calculated.

$$Z_i = \frac{Z_0}{2} \frac{\sinh(\gamma l)}{\cosh(\gamma l) - 1}$$

(4.19)

This can be rearranged

$$Z_i = \frac{Z_0}{2} \frac{1 - j \tanh\left(\frac{\alpha l}{2}\right) \tan\left(\frac{\beta l}{2}\right)}{\tanh\left(\frac{\alpha l}{2}\right) + j \tan\left(\frac{\beta l}{2}\right)}$$

(4.20)

Where $\alpha$ represents the line loss and $\beta$ the propagation factor as normal.

Around resonance the following are assumed ($v_p$ is the phase velocity of the line, $n \omega_0 = \omega_h$ is a resonant frequency and $\Delta \omega$ is a small frequency offset)

$$\omega = n \omega_0 + \Delta \omega$$

$$\beta = \frac{\omega}{v_p}$$

(4.21)

$$\beta l = \frac{n \omega_0 l}{v_p} + \frac{\Delta \omega l}{v_p}$$

At $\omega_0$ (4.15) holds and

$$l = n \lambda_g = n. \frac{2 \pi v_p}{\omega_0} = \frac{2 \pi v_p}{\omega_0}$$

(4.22)

Therefore

$$\beta l = 2n \pi \left(1 + \frac{\Delta \omega}{n \omega_0}\right)$$

(4.23)

Using small angle approximations for $\tan$ and $\tanh$ in (4.20) by assuming the line is low loss ($\alpha l$ small)

$$Z_i \approx \frac{Z_0}{2} \left(1 - \frac{\alpha l}{2} + \frac{2 n \pi \Delta \omega}{2 n \omega_0} \right)$$

$$Z_i \approx \frac{\alpha l}{2} + j \frac{2 n \pi \Delta \omega}{2 n \omega_0}$$

(4.24)

In [38] the following expression is given for a parallel LCR circuit
\[ Q = \omega_0 RC \]
\[ \omega_0 = \frac{1}{\sqrt{LC}} \]
\[ Z_i \approx \frac{R}{1 + 2jQ\Delta\omega/\omega_0} \]
\[ Z_i \approx \frac{1}{1/R + 2jC\Delta\omega} \]

Hence the following can be inferred

\[ R \rightarrow \frac{\alpha l}{Z_0} \]
\[ C \rightarrow \frac{\pi}{Z_0 \omega_0} = \frac{n\pi}{Z_0 \omega_n} \]
\[ L \rightarrow \frac{1}{\omega_n^2 C} = \frac{Z_0}{n^2 \omega_n \pi} \]

It can be noticed that these expressions are identical to those of Hsieh and Chang except that \( L \) decreases with frequency. This trivial extension to [20] makes the model much more versatile. Since \( \omega_0 \) is \( n \omega_0 \) the Q of the ring can be calculated for higher modes.

\[ Q = \frac{Rn}{Z_0} \]

This is very useful as it shows that (ignoring variations of \( R \) with frequency) that the Q increases at higher modes (frequencies), and with increasing \( Z_0 \) (decreasing ring width). This provides a practical rule of thumb for making higher Q resonators.

### 4.5 Losses

There are three loss mechanisms which contribute to the resistance, \( R \).

1. Dielectric loss
2. Conductor loss
3. Radiation loss

Two methods of analysis have been investigated for working out these loss factors. The dielectric and conductor losses can be estimated using microstrip design equations. The radiation loss can only be assessed by an electromagnetic analysis. The electromagnetic model can also estimate losses in the conductor and dielectric.

The dielectric loss (in Np/m) in a microstrip line can be calculated from the attenuation constant of a TEM wave [38]

\[ \alpha_d = \frac{k \tan \delta_{eff}}{2} = \frac{\beta \tan \delta_{eff}}{2} \]

Hence
\[
\alpha_d = \frac{2\pi f}{c} \sqrt{\varepsilon_{\text{eff}}} \cdot \frac{\tan \delta_{\text{eff}}}{2}
\] (4.29)

However, some of the electric field lies within the air, which is assumed to be lossless, therefore the effective loss tangent can be calculated from the material loss tangent by multiplying by a filling factor. This filling factor, derived by Wheeler is presented in [9] and gives the ratio of the electric energy stored in the dielectric to that stored in air for microstrip.

\[
p = \frac{W_d}{W} = \frac{\varepsilon_r}{\varepsilon_{\text{eff}}} \frac{(\varepsilon_{\text{eff}} - 1)}{(\varepsilon_r - 1)}
\] (4.30)

The dielectric attenuation constant therefore becomes

\[
\alpha_d = \frac{2\pi f}{c} \sqrt{\varepsilon_{\text{eff}}} \cdot \frac{\tan \delta_{\text{eff}}}{2} \cdot p
\]

\[
= \frac{\pi f}{c} \cdot \tan \delta_{\text{eff}} \cdot \frac{\varepsilon_r}{\varepsilon_{\text{eff}}} \frac{(\varepsilon_{\text{eff}} - 1)}{(\varepsilon_r - 1)}
\] (4.31)

This is commonly converted to dB/m in the literature and results in

\[
\alpha_d = 27.3 \frac{f}{c} \tan \delta_{\text{eff}} \cdot \frac{\varepsilon_r}{\sqrt{\varepsilon_{\text{eff}}}} \frac{(\varepsilon_{\text{eff}} - 1)}{(\varepsilon_r - 1)} \text{ dB/m}
\] (4.32)

The calculation of microstrip conductor loss is much more complicated, and can be rather inaccurate. Due to conductors having a finite conductivity, an electric field exists within the metal, and current flows inside the conductor. The conductor loss calculations make use of the skin depth:

\[
\delta = \sqrt{\frac{2}{\omega \mu \sigma}}
\] (4.33)

Which is the depth into the conductor where the current density drops to 1/e of the surface current density. Use is also made of the surface resistance, which is the resistance of a square of conductor, assuming that the current flows only to a depth of \(\delta\).

\[
R_s = \frac{1}{\sigma \delta} = \sqrt{\frac{\pi \mu}{\sigma}}
\] (4.34)

Unfortunately, except in the case of very wide microstrip lines, the surface current distribution is not uniform, so this equation cannot directly be used to calculate the loss of the microstrip. Wheeler noted that the current flowing inside the conductor would produce magnetic fields, and hence increased inductance in the microstrip line. It can be shown that the reactance produced by this increased inductance is equal to the series surface impedance [9], so if the increase in inductance could be calculated, then the resistive loss would also be known. This is the basis of Wheeler’s incremental inductance rule which has been applied by Pucel, Masse and Hartwig [39] and Hammerstad to estimate the loss due to finite metal conductivity. Whilst this model is widely
used, it has several shortcomings. It is only valid on conductors thicker than about $3\delta - 5\delta$ and it does not account for imperfect conductor finishes such as roughness. The conductor loss is proportional to the surface resistance, but also strongly dependent on the geometry of the microstrip.

Hammerstad [6] has published an approximate formula for accounting for the surface roughness effects, where $\Delta$ is the RMS surface roughness. The overall accuracy of this equation is unknown as it is based upon numerical analysis on periodic rectangular or triangular grooves in the conductor [41].

$$R_s' = R_s \left[1 + \frac{2}{\pi} \arctan \left(1.4 \frac{\Delta^2}{\delta^2}\right)\right]$$  \hspace{1cm} (4.35)

Hammerstad produced the following equations for calculating the conductor loss of a microstrip line [9]

$$\alpha_c = \begin{cases} 
\frac{1.38R_s' \left(32 - (w/h)^2\right)}{2(32 + (w/h)^2)} & w/h \leq 1 \\
\frac{6.12 \times 10^{-5} \Delta Z_0 \varepsilon_{\text{eff}}(f)}{h} \left[\frac{w}{h} + \frac{6h}{w} \left(1 - \frac{h}{w}\right)^5 \right] + 0.08 \left[1 + \frac{h}{w} \left(1 + \frac{\partial w}{\partial r}\right)\right] & w/h \geq 1
\end{cases}$$  \hspace{1cm} (4.36)

In order to calculate the radiation loss, an electromagnetic analysis must be carried out. This is because the radiation is due to the curvature of the microstrip rather than some intrinsic property of the microstrip.

4.6 Electromagnetic far field analysis

To determine the individual loss contributions, it is necessary to look at the electromagnetic fields around the ring structure. The fields are found by solving Maxwell’s equations in cylindrical co-ordinates, using the cavity model. Whilst this is a significant simplification, Full wave analytical solutions including the effects of fringing fields around the ring, thick substrates and radiation are extremely complicated and offer little physical insight.

![Figure 8 - The microstrip ring under the cavity model](image)

Note that the periphery of the ring is bounded with vertical perfect magnetic conductors (PMC) and the top of the ring and ground plane is a perfect electric conductor (PEC). The width of the ring is
modified by the cavity model to account for the fringing fields and is frequency dependent as shown by (4.4). The dielectric constant inside the ring is also modified to $\varepsilon_{\text{eff}}(f)$.

To compute the electric (far) fields, it is convenient to change to spherical coordinates and neglect the height of the ring. The electric field across the vertical apertures on the inner and outer edges of the ring produce radiation. Using Huygens equivalence principle [22] these vertical electric aperture fields can be replaced by equivalent surface magnetic currents flowing around the edges of the ring.

\[ \rho = a_e \]
\[ \rho = b_e \]
\[ \theta \]
\[ \phi \]
\[ M_I \]
\[ M_O \]

**Figure 9 - Equivalent magnetic sheet current sources flowing around the ring**

Figure 9 shows the equivalent magnetic currents flowing around the inside ($M_I$) and outside ($M_O$) of the ring. The magnetic current sources are assumed to flow on the top of the ring with corresponding images in the ground plane. The exact location of these magnetic sheet current sources is some source of speculation. Bahl and Stuchly [17] assume them to be line sources located at $a_e$ and $b_e$ whereas Das, Das and Mathur [13] assumed them to have a uniform radial distribution and located between $a$ and $a-h$ (inner current) and $b$ and $b+h$ (outer current). Assuming the currents to be of the form of sheets rather than lines significantly complicates the analysis resulting in very complicated expressions. No analysis comparing the two techniques appears to have been carried out, but both seem to offer reasonable results. In reality, several simplifying assumptions in all of these electromagnetic analyses could quite easily nullify the extra effort in obtaining “exact” expressions. Although this width is arbitrary [17], it is commonly used and accounts for the fringing fields having an exponential decay with distance [12]. There are a pair of image currents flowing in the ground plane. In calculating the radiation fields, several simplifying assumptions will be made

1. The equivalent magnetic current distributions are uniform in the radial direction
2. The equivalent magnetic currents exist only in the regions $a-h < \rho < a$ and $b < \rho < b+h$
3. The equivalent magnetic current distribution varies only sinusoidally with $\phi$
4. The substrate is thin enough to neglect the phase difference between the magnetic currents and their images in the ground plane.

The magnetic current is found from
\[ \mathbf{M} = -\hat{n} \times \mathbf{E} \]  
\[ (4.37) \]

where \( \hat{n} \) is a normal unit vector. Since the cavity model assumes that only \( E_z \) electric field components exist, and the normal vector is \( \rho \)

\[ \mathbf{M}_o = -\hat{\rho} \times \mathbf{E}(\rho = b_c) \]
\[ = \hat{\rho} E_z(\rho = b_c) \]
\[ \mathbf{M}_1 = -\hat{\rho} \times \mathbf{E}(\rho = a_c) \]
\[ = \hat{\rho} E_z(\rho = a_c) \]  
\[ (4.38) \]

The analysis will be carried out assuming that the current sheets are distributed as defined by Das et al. The following analysis is based upon the method described in [22].

The authors of [22] quote for magnetic current sheets in a free space spherical coordinate system the far field electric potential can be found from

\[ F_\theta = -\frac{\varepsilon_0}{4\pi} \frac{e^{-jk_d}}{r} \cos \theta E_z \int_0^{2\pi} M_\phi(\rho, \phi') \sin(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} \rho d\rho d\phi' \]  
\[ (4.39) \]

\[ F_\phi = \frac{\varepsilon_0}{4\pi} \frac{e^{-jk_d}}{r} \int_0^{2\pi} M_\phi(\rho, \phi') \cos(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} \rho d\rho d\phi' \]  
\[ (4.40) \]

By assuming the radial component of the magnetic surface current is constant, the integral becomes significantly simplified.

At the inside edge

\[ F_\theta = \frac{\varepsilon_0}{4\pi} \frac{e^{-jk_d}}{r} \cos \theta E_z \int_0^{2\pi} \cos(\rho \phi') \sin(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} d\phi' \int_a^{a+h} \rho d\rho \]  
\[ (4.41) \]

\[ F_\phi = - \frac{\varepsilon_0}{4\pi} \frac{e^{-jk_d}}{r} E_z \int_0^{2\pi} \cos(\rho \phi') \cos(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} d\phi' \int_a^{a+h} \rho d\rho \]  
\[ (4.42) \]

The integral over \( \rho \) is simplified by assuming that the substrate is thin, so \( h^2 \) terms can be neglected.

The integral over \( \phi' \) is ugly, but has a short solution using the following identities [22]

\[ \int_0^{2\pi} \cos(\rho \phi') \cos(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} d\phi' = -2\pi j^{n+1} \cos n\phi J_n(k_0a \sin \theta) \]  
\[ (4.43) \]

and

\[ \int_0^{2\pi} \cos(\rho \phi') \sin(\phi' - \phi) e^{jk_c \rho \sin \theta \cos(\phi' - \phi)} d\phi' = 2\pi j^{n+1} \sin n\phi J_n(k_0a \sin \theta) \]  
\[ (4.44) \]

The Electric far fields are related to the potential via

\[ E_\phi = j\omega \eta_0 F_\theta \]
\[ E_\theta = -j\omega \eta_0 F_\phi \]  
\[ (4.45) \]

Resulting in

\[ E_\phi = -\frac{\omega \eta_0 \varepsilon_0 n e^{-jk_d}}{2} \frac{e^{-jk_d}}{r} \cos \theta E_z \int_0^{2\pi} \sin n\phi J_n(k_0a \sin \theta) \frac{J_n'(k_0a \sin \theta)}{k_0a \sin \theta} d\phi \]  
\[ (4.46) \]
\[ E_{az} = \frac{\omega \eta_0 \varepsilon_0}{2} e^{-jk_0r} \left. E_z \right|_{\varphi=\theta} \cdot j^n \cos n\phi J_n \left( k_0 a \sin \theta \right) \cdot \vartheta (4.47) \]

Since
\[ k_0 = \frac{\omega}{c} \]
\[ c = \frac{1}{\eta_0 \varepsilon_0} \quad (4.48) \]

\[ E_{az} = -j^n a \cdot h \cdot k_0 \frac{e^{-jk_0r}}{2r} \cdot \cos \theta \cdot \sin n\phi \frac{J_n \left( k_0 a \sin \theta \right)}{k_0 a \sin \theta} \cdot E_z \left|_{\varphi=\theta} \right. \quad (4.49) \]

\[ E_{az} = j^n a \cdot h \cdot k_0 \frac{e^{-jk_0r}}{2r} \cdot \cos n\phi J_n \left( k_0 a \sin \theta \right) \cdot E_z \left|_{\varphi=\theta} \right. \quad (4.50) \]

These expressions match those in [22], except for a factor of two. This arises as the authors assume that the image current flowing in the groundplane adds in phase. This assumption is not made by Das et al. [13] who include an array factor. Since this current work is concerned with thin microstrip circuits rather than antennas, the reasonable assumption that the substrate is thin and there is no phase shift across it is made. Thus the groundplane array factor is simply 2.

The derivation above only considered the magnetic currents due to the inner edge. There are similar magnetic currents flowing around the outer edge, and the analysis is identical, with the exception that the fields are reversed due to the opposite direction of the current flow.

The overall radiation pattern can then be calculated by considering the vector sum of the fields from the two edges.

Using the Wronskian property of Bessel functions [40]
\[ J_n (q) Y_n (q) - J_n (q) Y_n (q) = \frac{2}{\pi q} \quad \forall q \quad (4.51) \]

And the resonance equation for the ring resonator
\[ J_n \left( k_{mn} a \right) Y_n \left( k_{mn} b \right) - J_n \left( k_{mn} b \right) Y_n \left( k_{mn} a \right) = 0 \quad (4.52) \]

The expression for \( E_Z \) was presented earlier
\[ E_Z = E_0 \left[ J_n \left( k_{mn} \rho \right) Y_n \left( k_{mn} a \right) - J_n \left( k_{mn} a \right) Y_n \left( k_{mn} \rho \right) \right] \cos n\phi \quad (4.53) \]

Therefore the magnitude of the electric field at the edges of the ring can be calculated. Note that the azimuth variation of the electric field is not required here, as the surface magnetic current already includes this factor.
\[ E_{az} \left|_{\varphi=\theta} \right. = E_0 \left[ J_n \left( k_{mn} a \right) Y_n \left( k_{mn} a \right) - J_n \left( k_{mn} a \right) Y_n \left( k_{mn} a \right) \right] \quad (4.54) \]
\[ E_z |_{y=b} = E_0 \left[ J_n(k_{nm} b)^* Y_n(k_{nm} a) - J_n(k_{nm} a)^* Y_n(k_{nm} b) \right] \]

\[ = \frac{2E_0}{\pi k_{nm}} \frac{J_n(k_{nm} a)}{J_n(k_{nm} b)} \]  

(4.55)

Therefore the total electric far fields can be calculated

\[ E_\theta = j^n h k_0 \frac{2E_0}{\pi k_{nm}} \frac{e^{-jk_0 r}}{r} \cos n\phi \left[ J_n(k_0 a \sin \theta) - J_n(k_0 b \sin \theta) \frac{J_n(k_{nm} a)}{J_n(k_{nm} b)} \right] \]  

(4.56)

\[ E_\phi = -j^n n h k_0 \frac{2E_0}{\pi k_{nm}} \frac{e^{-jk_0 r}}{r} \cos \theta \sin n\phi \left[ J_n(k_0 a \sin \theta) - J_n(k_0 b \sin \theta) \frac{J_n(k_{nm} a)}{k_0 \sin \theta \ J_n(k_{nm} b)} \right] \]  

(4.57)

These expressions are identical to those in [22], as the groundplane array factor of 2 has now been included.

The radiated power can be calculated by integrating the far field radiation pattern over a hemisphere (assuming an infinite groundplane).

\[ P_r = \frac{1}{2\eta_0} \int_0^{\phi=2\pi} \int_0^{\theta=\pi/2} \left( |E_\theta|^2 + |E_\phi|^2 \right)^2 \sin \theta d\theta d\phi \]  

(4.58)

The integral over \( \phi \) is straightforward but the integral over \( \theta \) must be evaluated numerically.

\[ P_r = \frac{1}{2\eta_0} \left( \frac{2E_0 k_0 h}{\pi k_{nm}} \right)^2 \]

\[ \sin \theta \cos^2 \theta \cos n\phi \left[ J_n^*(k_0 a \sin \theta) - J_n^*(k_0 b \sin \theta) \frac{J_n(k_{nm} a)}{J_n(k_{nm} b)} \right]^2 \]

(4.59)

\[ P_r = \frac{1}{2\eta_0} \left( \frac{2E_0 k_0 h}{\pi k_{nm}} \right)^2 \pi \]

\[ \sin \theta \left[ J_n^*(k_0 a \sin \theta) - J_n^*(k_0 b \sin \theta) \frac{J_n(k_{nm} a)}{J_n(k_{nm} b)} \right]^2 \]

(4.60)

Garg et al [22] and [21] refer to the integral as \( I_1 \). Relating \( k_0 \) and \( k_{nm} \) by the effective dielectric constant results in:

\[ P_r = \frac{2h^2 E_0^2}{\eta_0 \pi \varepsilon_{eff}} I_1 \]  

(4.61)
Note that this expression is different to that presented by Garg et al [22] and [21]. Their expression appears to contain a typographical error and is dimensionally incorrect.

The loss in the dielectric can be calculated from the fields beneath the ring.

\[ P_d = \frac{\omega \varepsilon''}{2} \iint_E \left| E_z \right|^2 dV \]  

(4.62)

Where \( \varepsilon'' \) is the imaginary (loss) part of the permittivity. This is deduced from the definition of \( \tan\delta \)

\[ P_d = \frac{\omega \varepsilon_0 \varepsilon_r \tan \delta E_0^{'^2}}{2} \]

\[ \int_{b=0}^{b=0} \int_{\phi=0}^{\phi=2\pi} \int_{z=0}^{z=b} \left[ J_n'(k_{nm}\rho)Y_n'(k_{nm}\rho) - J_n'(k_{nm}\rho)Y_n'Y_n(k_{nm}\rho) \right]^2 \cos^2 n\phi \rho d\phi d\rho dz \]  

(4.63)

\[ P_d = \frac{\omega \varepsilon_0 \varepsilon_r \tan \delta E_0^{'^2}}{2} \int_{\phi=0}^{\phi=0} \int_{\rho=a}^{\rho=b} \left[ J_n'(k_{nm}\rho)Y_n'(k_{nm}\rho) - J_n'(k_{nm}\rho)Y_n'Y_n(k_{nm}\rho) \right]^2 \rho d\rho \]  

(4.64)

The integral is difficult, but after several steps it can be shown that at resonance [22].

\[ P_d = \frac{\omega \varepsilon_0 \varepsilon_r \tan \delta E_0^{'^2} \pi}{2} \int_{\phi=0}^{\phi=0} \int_{\rho=a}^{\rho=b} \left[ J_n'(k_{nm}\rho)Y_n'(k_{nm}\rho) - J_n'(k_{nm}\rho)Y_n'Y_n(k_{nm}\rho) \right]^2 \rho d\rho \]  

(4.65)

The conductor loss is found from integrating the surface current density over the area of the ring conductor. Note that the conductor loss is doubled as it is assumed that an equal loss occurs in the ground plane. \( R_S \) is the surface resistance as before.

\[ P_c = \frac{2R_S}{2} \int_{a}^{b} \left[ \left| J_\phi \right|^2 + \left| J_\rho \right|^2 \right] \rho d\rho \]  

(4.66)

Expressions for the current density have not been derived, however they follow directly from the magnetic field:

\[ J_\phi = -H_\rho = \frac{j\omega \varepsilon_0 E_0}{k^2} \frac{\partial E_z}{\partial \phi} = \frac{j\omega \varepsilon_0 E_0}{k^2} \frac{1}{\mu_0^2} \frac{1}{\rho} \frac{1}{\mu_0} \left[ J_n'(k_{nm}\rho)Y_n'(k_{nm}\rho) - J_n'(k_{nm}\rho)Y_n'Y_n(k_{nm}\rho) \right] \sin n\phi \]

\[ J_\rho = H_\phi = \frac{j\omega \varepsilon_0 E_0}{k^2} \frac{\partial E_z}{\partial \rho} = \frac{j\omega \varepsilon_0 E_0}{k^2} \frac{1}{\mu_0^2} \frac{1}{\rho} \frac{1}{\mu_0} \left[ J_n'(k_{nm}\rho)Y_n'(k_{nm}\rho) - J_n'(k_{nm}\rho)Y_n'Y_n(k_{nm}\rho) \right] \cos n\phi \]  

(4.67)

Again, the integral is long however several simplifications can be applied at resonance and the resulting expression for \( P_c \) is [22]

\[ P_c = \frac{2R_S E_0^2}{\pi \omega^2 \mu_0^2} \left[ \frac{J_n'(k_{nm}\rho)}{J_n'(k_{nm}\rho)} \right] \left[ 1 - \frac{n^2}{k_{nm}^2 b^2} \right] \left[ 1 - \frac{n^2}{k_{nm}^2 a^2} \right] \]  

(4.68)

One point of interest is that the resonance condition has been defined based upon the effective ring width, whereas the limits of the conductor loss integral are taken over the physical ring width.
Strictly speaking, this means that several of the simplifications in the derivation based upon the ring characteristic equation at resonance do not apply.

At this stage, expressions for $P_C$, $P_R$ and $P_D$ have been defined, but all are all in terms of the electric field magnitude, $E_0$. If the ring is fed at the edge either directly or indirectly, then the voltage at the feed is given by

$$V_0 = hE_z|_{\rho=\infty, \phi=0} = hE_0 \left[ J_n \left( k_{nm} b_e \right) Y_n \left( k_{nm} a_e \right) - J_n \left( k_{nm} a_e \right) Y_n \left( k_{nm} b_e \right) \right]$$  \hspace{1cm} (4.69)

Therefore the input loss impedance due to dielectric, radiation and conductor loss can be calculated as the electric field magnitude term cancels.

$$R_D = \frac{V_0^2}{2P_D}$$

$$R_R = \frac{V_0^2}{2P_R}$$  \hspace{1cm} (4.70)

$$R_C = \frac{V_0^2}{2P_C}$$

Therefore all components in the ring equivalent circuit can be calculated. The accuracy of the electromagnetic equations for calculating the loss resistances due to the dielectric loss and conductor loss is questionable, because by definition, the cavity model assumes that all fields are contained within the cavity, and so the dielectric loss in particular will be over estimated. The conductor loss calculation also assumes that equal currents flow in the top and bottom layers of the circuit, in reality the current distribution will be different as the groundplane is much wider than the top conductor. Figure 10 shows the ring equivalent circuit, around resonance.

![Figure 10 – Equivalent circuit of ring close to resonance](image)

4.7 Experimental verification

In order to verify the equations, test rings have been fabricated on alumina and RT Duroid 5870. The experiments are described more fully in the publications, [24],[37] but are briefly summarised here.

The low dielectric constant of Duroid ($\varepsilon_r=2.33 \pm 0.02$) and thick substrate ensure that the radiation loss is quite significant. Two sets of resonators were constructed as this allowed their radiation properties to be measured. Table 1 gives the dimensions of the rings. The PCB process used was unable to produce small gaps, and has a poor dimensional tolerance. It was therefore
necessary to predistort the design by making the conductors slightly wider than desired to allow for undercutting errors which decreases the conductor widths. The ring dimensions where then measured with a travelling microscope. These dimensions are accurate to within 25\(\mu\)m. Note that models 2A and 2B are both solid disks rather than rings, i.e. their inner radii is zero. Apart from construction tolerances, board A and B are identical. The Duroid thickness is 1.575mm.

The predicted resonant frequency is calculated from (4.13).

<table>
<thead>
<tr>
<th>Ring</th>
<th>Gap size</th>
<th>Outside radius</th>
<th>Inside radius</th>
<th>Predicted resonant frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>500(\mu)m</td>
<td>17.99mm</td>
<td>16.97mm</td>
<td>2026MHz</td>
</tr>
<tr>
<td>2A</td>
<td>230(\mu)m</td>
<td>10.48mm</td>
<td>[Solid disk]</td>
<td>4669MHz</td>
</tr>
<tr>
<td>3A</td>
<td>230(\mu)m</td>
<td>7.68mm</td>
<td>6.09mm</td>
<td>5141MHz</td>
</tr>
<tr>
<td>4A</td>
<td>210(\mu)m</td>
<td>17.98mm</td>
<td>16.99mm</td>
<td>2026MHz</td>
</tr>
<tr>
<td>1B</td>
<td>530(\mu)m</td>
<td>17.95mm</td>
<td>16.99mm</td>
<td>2028MHz</td>
</tr>
<tr>
<td>2B</td>
<td>270(\mu)m</td>
<td>10.46mm</td>
<td>[Solid disk]</td>
<td>4677MHz</td>
</tr>
<tr>
<td>3B</td>
<td>260(\mu)m</td>
<td>7.67mm</td>
<td>6.10mm</td>
<td>5142MHz</td>
</tr>
<tr>
<td>4B</td>
<td>270(\mu)m</td>
<td>17.95mm</td>
<td>17.02mm</td>
<td>2027MHz</td>
</tr>
</tbody>
</table>

Figure 11 – Photograph of Duroid ring resonators

A wideband plot of ring B4 on Duroid return loss is shown in Figure 12. The first 10 resonances are clearly seen, although there are additional features on the plot.
It is thought that these additional features could be due to the effect of a higher order mode propagating in the feedline. Using the parallel plate wave guide model, it can be seen that a higher order mode is supported when the width of the waveguide is more than half a wavelength. The cut off frequency for this mode can be calculated from the planar waveguide model [4], by assuming that the dielectric is homogenous.

\[ w_{\text{eff}}(f) = \frac{h \eta_0}{Z_0(f)\sqrt{\varepsilon_{\text{eff}}(f)}} = \frac{\lambda_g}{2} \]  \hfill (4.71)

Using the fact that the dielectric is homogenous in the planar waveguide, the guide wavelength can be calculated at the cutoff frequency

\[ w_{\text{eff}}(f_{\text{co}}) = \frac{h \eta_0}{Z_0(f_{\text{co}})\sqrt{\varepsilon_{\text{eff}}(f_{\text{co}})}} = \frac{\lambda_g}{2} = \frac{c}{2f_{\text{co}}\sqrt{\varepsilon_{\text{eff}}(f_{\text{co}})}} \]  \hfill (4.72)

Hence

\[ f_{\text{co}} = \frac{cZ_0(f_{\text{co}})}{2h \eta_0} \]  \hfill (4.73)

Ignoring dispersion and assuming that the feedline impedance is still 50\(\Omega\) at cutoff, this cutoff frequency is calculated to be approximately 13GHz in 1.575mm thick Duroid. Therefore, there are multiple modes propagating in the microstrip feed line above 13GHz and the actual propagation in the feed line will be a hybrid mode. Because of the different field distribution of this wave, the impedance of this wave will be different to that of the quasi TEM microstrip mode, hence the feed line is no longer matched to the 50\(\Omega\) network analyser and could act as a resonator when it is a
multiple of half a wavelength long – This could be occurring at 18GHz as an extra dip is present in the return loss plot at this frequency. Normally higher order modes are avoided on microstrip as they have significant dispersion. It is important to note that the microstrip ring track is much narrower than the feed line, so even though the feed line is operating with hybrid mode propagation, the microstrip ring still operates in the fundamental quasi TEM microstrip mode.

The rings fabricated on alumina substrates used thick film silver conductors. The substrate used was Coorstek ADS96R, in 10mil and 25mil thicknesses. This substrate has a low loss tangent even at mm-wave frequencies. A Wiltron 3680 universal test jig was used to eliminate the need for soldered coaxial edge connectors, which have relatively poor performance at microwave frequencies. The use of this jig improves measurement repeatability.

![Image](Figure 13 – Photograph of Alumina ring resonator on Wiltron Universal test jig)

The alumina ring had the following parameters

<table>
<thead>
<tr>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor</td>
</tr>
<tr>
<td>Inside radius</td>
</tr>
<tr>
<td>Outside radius</td>
</tr>
<tr>
<td>Gap size</td>
</tr>
<tr>
<td>Feed width</td>
</tr>
<tr>
<td>Feed length</td>
</tr>
<tr>
<td>Substrate height</td>
</tr>
<tr>
<td>εᵣ</td>
</tr>
</tbody>
</table>

A wideband plot of return loss is shown in Figure 14 for the alumina ring on a 25mil substrate
Eight resonances can clearly be seen, with a small amount of residual mismatch, although this mismatch is much less significant than that on the Duroid ring resonators due to the use of a thinner substrate and narrower tracks. Using (4.73), the cut off frequency for the transverse microstrip mode is about 31GHz, so over the frequency range DC-25GHz, higher order modes are not excited.

4.8 Equivalent circuit fitting

In order to measure the Q factor (and hence loss) of the ring around each resonance, a narrow band sweep of 100-200MHz was taken of each circuit at each resonance. These data were then saved to disk, and imported in to Agilent ADS as a one port S parameter device. The circuit of Figure 15 was also created in ADS.

The gap between the microstrip feed and the ring has proved very difficult to model, and there appear to be no rigorous solutions to the problem. The problem with modelling the gap relates to the
fact that the fields are very concentrated in this region and the geometry of the gap consists of
different shaped conductors in a non-homogenous dielectric environment. It has been pointed out
that the gap looks similar to that formed between two open ended microstrip lines. Closed form
expressions exist for modelling this gap, but even these equations, based on the geometrically
simpler situation, are based on curve fitting. Although a full wave solution undoubtedly exists to
solving the field distribution between the microstrip and ring, the conceptual simplicity of an
equivalent circuit has great practical benefits. The most widely cited results for the coupling
between two equal collinear microstrip lines are from [19]-[34] gives curves (but not equations) for
asymmetric microstrips and [35] gives a mathematical full wave solution, but no equivalent circuit.
Yu and Chang [18] adjusted the equations from [19] to eliminate a discontinuity in the curves. The
model proposed by Yu and Chang consists of two capacitors which couple the transmission line to
the ring. A series coupling capacitor, C_g can be seen in the equivalent circuit, but the shunt
 capacitance, C_p has been absorbed into the electrical length of the transmission line feed. A
description of the parameters used in the circuit is given in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Optimisation Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Feed line length</td>
<td>Primarily defined by physical length but exact value unknown</td>
<td>Yes</td>
</tr>
<tr>
<td>Z_0</td>
<td>Feed line impedance</td>
<td>Determined using standard microstrip equations (with dispersion)</td>
<td>No</td>
</tr>
<tr>
<td>R_p</td>
<td>Shunt loss resistance</td>
<td>Value optimised to fit local non resonant losses around resonance</td>
<td>Yes</td>
</tr>
<tr>
<td>C_g</td>
<td>Coupling capacitance</td>
<td>Initial value calculated from [12] (assumes symmetrical microstrip) based on microstrip size and gap size</td>
<td>Yes</td>
</tr>
<tr>
<td>C</td>
<td>Ring equivalent capacitance</td>
<td>Calculated from (37), includes dispersive effects</td>
<td>No</td>
</tr>
<tr>
<td>L</td>
<td>Ring equivalent inductance</td>
<td>Calculated from (37), fixed to resonate at the frequency determined by electromagnetic equations</td>
<td>Yes</td>
</tr>
<tr>
<td>R_T</td>
<td>Ring loss resistance (total)</td>
<td>Calculated from (24), determined by electromagnetic equations, includes the three loss resistances in parallel</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The optimiser was then used to vary some of the component values in the equivalent circuit in a
systematic fashion to minimise the difference between the measured S_{11} data and the S_{11} data of
the equivalent circuit. When optimised, the values of the components in the equivalent circuit were
recorded.

The following plots show the predicted and measured total loss resistances. For the Duroid,
tanδ was assumed constant at 0.001 and a metal conductivity of 5.8*10^7 S/m, thickness 17um and
roughness 1.5um were assumed. For the alumina substrate, tanδ was assumed constant at 0.001 and
a metal conductivity of 6.3*10^7 S/m, thickness 10um and roughness 0.9um were assumed. In the
following plots, the dielectric loss and conductor loss has been predicted using the microstrip
equations (4.32) & (4.36), shown on the left, and annotated “$R_T$ (Theoretical) MS” as well as using the electromagnetic equations (4.70). The Radiation loss is calculated using (4.70).

Figure 16 - Loss resistance of the Alumina ring, with 635um substrate thickness

Figure 17 - Loss resistance of the Alumina ring, with 254um substrate thickness

Figure 18 - Loss resistance of the Duroid ring, with 530um gap

Figure 19 - Loss resistance of the Duroid ring, with 530um gap
4.9 Radiation measurements

An expression for the directivity of a ring antenna can be found in [17], which has been derived from the electric radiation fields (4.56) and (4.57)

\[ D = \frac{k_o^2 (V_o a_e - V_i b_e)^2}{240 P_r} \]  

(4.74)

Where \( V_o \) and \( V_i \) are the voltages at the outside and inside of the ring and can be found from (4.2), although \( V_o \) is already defined in (4.69). The directivity is weakly dependent on the width of the ring, the dielectric height and the dielectric constant. For the rings described in Table 1, rings 1 and 4 have a directivity of 7.7dB and ring 3 has a directivity of 7.6dB.

Antenna plate A and B (which are identical within production tolerances) were mounted on tripods in an anechoic chamber facing each other, separated by a distance of 2.1m. One was connected to a microwave signal generator, the other to a power meter and the transmission loss measured. By accounting for the free space path loss, cable loss, mismatch loss and directivity, it is possible to calculate the antenna gains. The difference between the antenna gain and the directivity is the radiation efficiency.

\[
\text{FSPL} = 20 \log_{10} \left( \frac{4 \pi d}{\lambda} \right) 
\]

(4.75)

Where \( d \) is the separation distance (2.1m).

Unfortunately, due to manufacturing tolerance and very sharp frequency response, the two pairs of antennas do not resonate at precisely the same frequency. This requires making the assumption that the radiation efficiency and radiation properties do not vary much about the resonant frequency, only the mismatch factor. The mismatch factor is calculated according to (4.76)

\[ M = 10 \log_{10} \left( 1 - \left| S_{11} \right|^2 \right) \]  

(4.76)

\( S_{11} \) has been measured for all of the ring structures around resonance. The frequency of the signal generator was adjusted to give the maximum signal on the power meter.

---

*Figure 20 – diagram of losses / gains in link budget calculation*
Table 4

<table>
<thead>
<tr>
<th>Ring</th>
<th>1 (Narrow track)</th>
<th>3 (Wide track)</th>
<th>4 (Narrow track)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (MHz)</td>
<td>1989</td>
<td>5000</td>
<td>1985</td>
</tr>
<tr>
<td>Cable Loss (dB)</td>
<td>-0.3</td>
<td>-1.4</td>
<td>-0.3</td>
</tr>
<tr>
<td>Mismatch factor - Ring A (dB)</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>Directivity – Ring A (dB)</td>
<td>7.7</td>
<td>7.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Free space path loss (dB)</td>
<td>-44.8</td>
<td>-52.9</td>
<td>-44.8</td>
</tr>
<tr>
<td>Directivity – Ring B (dB)</td>
<td>7.7</td>
<td>7.6</td>
<td>7.7</td>
</tr>
<tr>
<td>Mismatch factor – Ring B (dB)</td>
<td>-2.1</td>
<td>0</td>
<td>-0.6</td>
</tr>
<tr>
<td>Total Loss (dB)</td>
<td>-31.9</td>
<td>-39.3</td>
<td>-30.4</td>
</tr>
<tr>
<td>Measured Loss (dB)</td>
<td>-41</td>
<td>-44.8</td>
<td>-39.7</td>
</tr>
<tr>
<td>Excess loss (dB)</td>
<td>-9.1</td>
<td>-5.5</td>
<td>-9.3</td>
</tr>
<tr>
<td>Estimated radiation efficiency</td>
<td>35%</td>
<td>53%</td>
<td>34%</td>
</tr>
</tbody>
</table>

The measurements are shown in Table 4. Furthermore, it is assumed that each patch has the same efficiency, therefore the radiation efficiency of antenna 1 is -9.1/2dB = -4.55dB = 35%.

The results show that antenna 1 and 4 have approximately the same radiation efficiency. This is to be expected as the rings 1 and 4 are the same, only the coupling gap size is different. A most important observation is that the radiation efficiency is fairly reasonable, especially for the comparatively wide ring. It is important to stress for all three of these antennas that the radiation loss resistance is higher than either the conductor or dielectric loss. Therefore the commonly made assumption that radiation losses are negligible in the ring resonator are false in this case.

The radiation efficiency can be defined as the (space wave) radiation power divided by the total power dissipated. In the context of this work

$$\eta = \frac{P_R}{P_R + P_D + P_c} = \frac{R_T}{R_T}$$

(4.77)

It is therefore possible to calculate the radiation efficiency from the equations given in this report. Assuming nominal ring sizes for convenience, and using the microstrip equations for dielectric and conductor loss, this is shown in Table 5

Table 5

<table>
<thead>
<tr>
<th>Inside radius</th>
<th>Outside radius</th>
<th>R_T</th>
<th>R_R</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring 1,4 17mm</td>
<td>18mm</td>
<td>6.8kΩ</td>
<td>25.1kΩ</td>
<td>27%</td>
</tr>
<tr>
<td>Ring 3 6.1mm</td>
<td>7.7mm</td>
<td>2.02kΩ</td>
<td>2.5kΩ</td>
<td>81%</td>
</tr>
</tbody>
</table>

Using the electromagnetic equations for all of the loss factors gives slightly different results

Table 6

<table>
<thead>
<tr>
<th>Inside radius</th>
<th>Outside radius</th>
<th>R_T</th>
<th>R_R</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring 1,4 17mm</td>
<td>18mm</td>
<td>9.07kΩ</td>
<td>25.1kΩ</td>
<td>36%</td>
</tr>
<tr>
<td>Ring 3 6.1mm</td>
<td>7.7mm</td>
<td>2.13kΩ</td>
<td>2.5kΩ</td>
<td>85%</td>
</tr>
</tbody>
</table>

It can be seen that the measured radiation efficiency for rings 1 and 4 is in very agreement with the theoretical values, but that of ring 3 is not as close. One explanation for this might be that (4.74)
only provides an estimate of the directivity of the ring, whereas the actual antenna has a large metallic feedline in close proximity with the ring. It is reasonable to assume that this will somehow affect the radiation pattern (and directivity), but its effects have not been included. The feedline is electrically much larger at 5GHz where ring 3 was measured, so this may go some way towards explaining the discrepancy. The size of the feedline in comparison with the ring can clearly be seen in the photograph of the rings (Figure 11).
An important conclusion that can be drawn from the radiation efficiency measurements is that the reasonable agreement between measured radiation efficiencies and predicted efficiencies shows that the ratio $R_T/R_R$ is approximately correct, which provides further evidence for the usefulness of the circuit model and the validity of the equations.
5 Future work

5.1 Resonators

Work to date has concentrated on the microstrip ring resonator at modest microwave frequencies. However there are situations when the use of microstrip is inappropriate. One of the disadvantages of microstrip is that the fields are concentrated in the dielectric, and at millimetre wave frequencies, the substrate loss may become unacceptably high. Microstrip also supports higher order modes with the second mode after the Q-TEM dominant mode being TE_{10} where there is a reversal of the electric field across the width of the strip. The propagation characteristics of these higher order modes in general will differ from those of the Q-TEM mode, so the microstrip will suffer more serious dispersive effects. Additional higher order modes propagate when the substrate is no longer electrically thin which requires the substrate to be very thin at high frequencies, which may result in physical difficulties in fabricating the line. Other types of line such as coplanar waveguide and slot line do not require thin substrates, and may be easier to fabricate. The use of stripline may also offer advantages as the homogenous dielectric means that it supports a pure TEM mode and this fundamental mode does not suffer from dispersion. Although stripline with infinite groundplanes cannot radiate, it does support parallel plate modes which can cause energy to leak away from discontinuities in the line. Stripline seems like a good method of measuring dielectric properties, as only two loss mechanisms need to be considered, although it suffers from higher order modes in the same way as microstrip. Additional difficulties may arise due to a trapped air layer in the lamination process, and the top and bottom groundplanes must be connected together to ensure that unwanted odd modes cannot be setup.

Coplanar lines such as CPW and slotline offer the highest frequency response, as the wave is guided along the surface of the substrate between closely spaced conductors. Air bridges may be necessary to prevent excitation of unwanted modes in isolated metal islands.

It is proposed that the use of ring resonators be extended into different types of planar transmission line structures to see if accuracy can be improved, and in particular to extend the frequency range of dielectric characterisation to higher frequencies where microstrip may not be suitable.

5.2 Conductor loss

As microwave circuits applications are created for increasingly high frequencies, losses due to the surface finish of the conductor becomes increasingly important. Once the surface feature dimensions approach the skin depth, the loss increases dramatically. Whilst methods exist for
experimentally measuring these losses (including of course ring resonators), the underlying theory
has not been studied in detail. Initial literature survey results have shown that all commonly cited
design equations for planar transmission lines use a simple curve fitted expression to account for the
surface roughness, however close examination of the expression shows that many drastic
simplifications and assumptions have been made in the derivation of the formula.

The original calculations of conductor loss were carried out by Morgan in 1948 [41] and
considered losses in waveguides at frequencies up to 24GHz. Through rigorous electromagnetic
methods, Morgan calculates the losses in metal with periodic ridges both parallel and perpendicular
to the direction of current flow. Due to the complexity of the mathematical equations, numerical
calculations could only be carried out for ridges of square or triangular section. Although the
mathematical method is explained, the integrals were calculated numerically and few results were
presented. Experimental confirmation of the results was not given. On the basis that the losses with
triangular grooves were comparable with losses with square grooves, the research concluded that
the exact shape of the surface was not critical. Despite the lack of experimental verification, a curve
fitted expression (based upon a curve sketched between five points) is still widely used today. At
millimetre wave frequencies, the drastic simplifications of this theory may no longer apply.

A new method of calculating conductor loss is proposed based upon the mathematics of
Morgan. Rather than using a simple one dimensional approximation to the surface roughness, it is
proposed that the surface can be modelled as an infinite summation of sinusoidal components with
variations over two dimensions. Using the spatial Fourier transform, any surface can be expressed
as a summation of appropriately weighted sinusoids. Because the interaction between the metal and
the electromagnetic fields is linear, it seems reasonable that the loss due to each sinusoidal
component can be individually calculated, the overall behaviour of the metal is found by
superposition. By measuring the surface roughness of a metal surface with a profile meter or atomic
force microscope, then taking the Fourier transform of the data, it may be possible to make
statistical generalisations about the frequency spectrum of typical surfaces.

It appears that this approach to determining conductor loss is new, and there is potential to
significantly further the knowledge on the behaviour of metals at millimetre wave frequencies.
### 6 Future Work Plan

The proposed work packages are presented below, with the timing broken down into quarterly periods.

<table>
<thead>
<tr>
<th>Work package</th>
<th>Resources</th>
<th>Q5 10/06</th>
<th>Q6 1/07</th>
<th>Q7 4/07</th>
<th>Q8 7/07</th>
<th>Q9 10/07</th>
<th>Q10 1/08</th>
<th>Q11 4/08</th>
<th>Q12 7/08</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Review background theory &amp; keep up to date with new developments</td>
<td>Library / Journal papers</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>1 Transfer viva</td>
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<tr>
<td>2. Microstrip Ring resonators</td>
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<td></td>
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</tr>
<tr>
<td>2.1. Design &amp; fabricate &amp; test Rings for 140-220GHz</td>
<td>Computer simulation, printed circuit fabrication, Network analyser</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>2.2. Develop, fabricate &amp; test differential ring technique to separate</td>
<td>Computer simulation, printed circuit fabrication, Network analyser</td>
<td>X</td>
<td>X</td>
<td></td>
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<tr>
<td>conductor and dielectric losses</td>
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<tr>
<td>2.3. Analyse microstrip results and publish a guidelines document</td>
<td>Computer simulation</td>
<td>X</td>
<td>X</td>
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<td></td>
</tr>
<tr>
<td>3. Theoretical study into conductor loss</td>
<td>Computer simulation, research</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Stripline (Triplate) Ring resonators</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>4.1 Develop theory for the resonators</td>
<td>Computer simulation, research</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>4.2 Design a practical test circuit &amp; measure performance</td>
<td>Computer simulation, printed circuit fabrication, Network analyser</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>4.3 Critically compare stripline and microstrip resonators &amp; document</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
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<tr>
<td>5. Exotic resonators for even higher frequencies</td>
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<tr>
<td>5.1 Develop models for surface guided wave resonators (eg CPW &amp; Slot line)</td>
<td>Computer simulation, research</td>
<td>X</td>
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<td>5.2 Produce a practical test circuit and measure over 140-220GHz</td>
<td>Computer simulation, printed circuit fabrication, Network analyser</td>
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<td>6 Documentation</td>
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<td>6.1 Comparison of different techniques to independently measure circuits</td>
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<td>6.2 Write up thesis &amp; examination</td>
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6.1 Task breakdown

The work plan shows 7 different work packages.

Task 0
The background reading is an ongoing task and involves keeping up to date with current advancements in the field of planar circuit design. This task does not require any facilities which are not currently unavailable.

Task 1
This report forms the basis of the PhD transfer process, and this task should be completed by the end of November 2006.

Task 2
This task involves applying the existing microstrip theory already developed into a series of practical experiments. These experiments will be designed in order to produce specific information about the properties of the polymer substrate. The ultimate objective will be to produce charts showing how the material behaves across its useful microwave frequency range. By measuring the properties of rings with different geometries, it is hoped that the dielectric and conductor loss factors can be separated, so that the effects of the conductor loss may be excluded. The rings will be designed in a way that minimises radiation loss, as this is difficult and time consuming to measure accurately. Because the polymer will need a high resolution conductive pattern to be etched upon it, some work in the clean room will be necessary in order to perfect the practical procedure. It is thought that the department already possesses adequate facilities for etching copper patterns with approximately 20µm resolution, so this is not expected to involve any significant difficulty. For test circuits developed in the 140-220GHz (\(\lambda_0=2.1-1.3\text{mm}\)) frequency range, it may be necessary to make physical measurements on the circuits to ensure that the test patterns are etched accurately. Instrumentation exists within the group to carry out the microwave measurements, however the analyser requires routine maintenance. There are other tasks which can easily be scheduled if the analyser is temporarily unavailable. This work is of practical interest, and it is likely that a tutorial style paper could be published that improves the accessibility of the measurement technique to non-specialists.

Task 3
During the literature study of the behaviour of microstrip circuits, it has become apparent that practical conductors exhibit less than ideal behaviour at very high frequencies. When the surface roughness macroscopic features of the metal approach the skin depth, the loss due to the finite conductivity becomes difficult to calculate. At 10GHz the skin depth of copper is about 800nm, and at 100GHz, it is only 250nm. Without special surface treatment (e.g. polishing), the conductor
roughness will significantly affect the loss seen in the circuit. Unfortunately, in a microstrip circuit, the underneath surface of the conductor is inaccessible as it is adhered to the substrate, so cannot be treated. This causes problems, as the resonator technique cannot directly distinguish between the loss due to the conductor and that due to the substrate. Therefore poor knowledge of the conductor loss translates to uncertainty in the dielectric loss.

A new method of calculating the conductor loss has been proposed and it is considered worthwhile investigating its possible benefits. If this work is successful, it would almost certainly yield significant new publishable material, and would support the existing program of work.

Task 4
This task focuses on developing resonators using a stripline (triplate) technique. This type of transmission line is similar to microstrip, but with an additional layer of dielectric and a top ground plane. The main advantage of stripline is that it supports a pure TEM mode, therefore does not suffer from dispersion. In principle this makes it ideal for measuring dielectric properties, however in practice stripline is constructed by a lamination process which introduces inhomogeneity where the layers meet. The absence of radiation, and the simpler design equations may however offer advantages in dielectric characterisation over microstrip. The department possesses facilities suitable for laminating multilayered circuits, and testing can be carried out on the existing network analysers.

Task 5
At frequencies where it is no longer feasible to make electrically thin substrates, different types of transmission lines are used. Instead of constraining the electric field within the substrate, guided wave structures such as coplanar waveguide (CPW) and slotline exist which support waves that primarily propagate along the surface of the substrate, guided by metal strips. These waveguides do not support TEM modes, and so are dispersive, however their operating frequency can be very high as it is possible to etch very fine conductors onto the substrate, so higher order modes occur at much higher frequencies than the fundamental mode. This task will concentrate on designing resonators using surface wave guide transmission lines.

Task 6
This task involves documenting the various stages of the research. To date, this documentation has been completed as the work has been carried out, as this has proven to be a useful method of clarifying the thoughts behind the work. This task will be ongoing.
7 References


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[40] M. Abramowitz, I. A. Stegun, “Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables”, Edited by: U.S. Department of Commerce